THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2009**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) Estimate the value of $57.2 \times 110 / 2.146 \times 46.9$ correct to one (1) significant figure

$57.2 \approx 60$ (1 s.f.)

$110 \approx 100$ (1 s.f.)

$2.146 \approx 2$ (1 s.f.)

$46.9 \approx 50$ (1 s.f.)

$60 \times 100 / (2 \times 50) = 6000 / 100 = 60$

Answer: 60

(b) Express 1.86 as an improper fraction in its simplest form

$1.86 = 186 / 100$

$= 93 / 50$ (divide by 2)

Answer: 93/50

2. (a) Solve for y if $(1/9)^{12y} (1/3)^{-4+y} = 27^{-3+y}$

Rewrite bases: $1/9 = 9^{-1} = (3^2)^{-1} = 3^{-2}$, $1/3 = 3^{-1}$, $27 = 3^3$

$(1/9)^{12y} = (3^{-2})^{12y} = 3^{-24y}$

$(1/3)^{-4+y} = (3^{-1})^{-4+y} = 3^{4-y}$

$27^{-3+y} = (3^3)^{-3+y} = 3^{-9+3y}$

Left: $3^{-24y} \times 3^{4-y} = 3^{-24y+4-y} = 3^{-25y+4}$

Right: 3^{-9+3y}

Exponents: $-25y + 4 = -9 + 3y$

$-25y - 3y = -9 - 4$

$-28y = -13$

$y = 13/28$

Answer: $y = 13/28$

(b) Simplify the expression $\sqrt{5} / \sqrt{11} - 3 + \sqrt{2} / \sqrt{22} + 3\sqrt{2}$

$\sqrt{5} / (\sqrt{11} - 3)$: Rationalize:

$\sqrt{5} (\sqrt{11} + 3) / (\sqrt{11} - 3)(\sqrt{11} + 3) = (\sqrt{5} (\sqrt{11} + 3)) / (11 - 9) = (\sqrt{55} + 3\sqrt{5}) / 2$

$\sqrt{2} / (\sqrt{22} + 3\sqrt{2})$: $\sqrt{22} = \sqrt{(11\times2)} = \sqrt{11}\,\sqrt{2}$

$\sqrt{2}(\sqrt{22} - 3\sqrt{2}) / (\sqrt{22} + 3\sqrt{2})(\sqrt{22} - 3\sqrt{2}) = (\sqrt{2}(\sqrt{11}\,\sqrt{2} - 3\sqrt{2})) / (22 - 9\times2) = (\sqrt{22}\,\sqrt{2} - 3\times2) / (22 - 18) = (2\sqrt{11} - 6) / 4 = (\sqrt{11} - 3) / 2$

Combine: $(\sqrt{55} + 3\sqrt{5}) / 2 + (\sqrt{11} - 3) / 2 = (\sqrt{55} + 3\sqrt{5} + \sqrt{11} - 3) / 2$

Answer: $(\sqrt{55} + 3\sqrt{5} + \sqrt{11} - 3) / 2$

3. In the figure drawn below, find the number of elements in sets:

(a) $A \cap (B \cup C')$

Assume sets A, B, C with elements as follows (common Venn diagram setup):

$A = \{a, b, c, d, g, h\}$, $B = \{b, d, e, f, g, i\}$, $C = \{c, e, f, g, j\}$

$C' = \{a, b, d, h, i\}$

$B \cup C' = \{b, d, e, f, g, i, a, h\}$

$A \cap (B \cup C') = \{a, b, d, g, h\}$

Number of elements = 5

Answer: 5

(b) $(A' \cap B') \cup (B \cup C')$

$A' = \{e, f, i, j\}$, $B' = \{a, c, h, j\}$

$A' \cap B' = \{j\}$

$B \cup C' = \{b, d, e, f, g, i, a, h\}$

$(A' \cap B') \cup (B \cup C') = \{j\} \cup \{b, d, e, f, g, i, a, h\} = \{a, b, d, e, f, g, h, i, j\}$

Number of elements = 9

Answer: 9

4. (a) Given vectors $a = i + 3j$, $b = 5i - 2j$ and $c = 3a - 4b$, find a unit vector in the direction of vector c

$a = i + 3j$, $b = 5i - 2j$

$c = 3a - 4b = 3(i + 3j) - 4(5i - 2j) = (3i + 9j) - (20i - 8j) = -17i + 17j$

Magnitude of $c = \sqrt{((-17)^2 + 17^2)} = \sqrt{(289 + 289)} = \sqrt{578} = 17\sqrt{2}$

Unit vector $= c / |c| = (-17i + 17j) / (17\sqrt{2}) = (-i + j) / \sqrt{2} = (-1/\sqrt{2}, 1/\sqrt{2})$

Answer: $(-1/\sqrt{2}, 1/\sqrt{2})$

5. (b) The point A (5, -7) is the vertices of the right angle of a right angled triangle whose hypotenuse lies along the line $6x - 13y = 0$ C (x, y) is a vertex of the triangle. Find the remaining vertex B (0, -3)

Line $6x - 13y = 0 \rightarrow y = (6/13)x$

Triangle ABC, right angle at A(5, -7), B(0, -3), C(x, y) on the line.

$AB \perp AC$ (right angle at A).

Vector AB = (0-5, -3-(-7)) = (-5, 4)

Vector AC = (x-5, y-(-7)) = (x-5, y+7)

Dot product $AB \cdot AC = 0$:

$(-5)(x-5) + 4(y+7) = 0$

$-5x + 25 + 4y + 28 = 0$

$-5x + 4y + 53 = 0$

C on line: $y = (6/13)x$

Substitute: $-5x + 4(6/13)x + 53 = 0$

$-5x + (24/13)x + 53 = 0$

$(-65x + 24x) / 13 + 53 = 0$

$-41x + 689 = 0$

$x = 689/41 = 16.8$

$y = (6/13)(689/41) = 96/41 \approx 2.34$

C = (689/41, 96/41)

Answer: C = (689/41, 96/41)

5. (a) A circle of radius 10 units is circumscribed by a right-angled isosceles triangle. Find the lengths of the sides of the triangle and hence perimeter (all in decimal places)

Right-angled isosceles triangle, hypotenuse = diameter = $2 \times 10 = 20$

Equal legs x: $x^2 + x^2 = 20^2 \rightarrow 2x^2 = 400 \rightarrow x^2 = 200 \rightarrow x = \sqrt{200} = 10\sqrt{2} \approx 14.14$

Perimeter = 14.14 + 14.14 + 20 = 48.28

Answer: Legs = 14.14, Hypotenuse = 20, Perimeter = 48.28

(b) In the figure below DE is parallel to BC, AD = 6 cm, BD = 3 cm DE = 4 cm, and ABC = 90°. Calculate

(i) the length of BC

Triangles ADE and ABC are similar (AA similarity).

AD/AB = DE/BC

AB = AD + BD = 6 + 3 = 9

6/9 = 4/BC

BC = 4 × 9 / 6 = 6 cm

Answer: BC = 6 cm

(ii) the ratio AE / AC

AE/AC = DE/BC (similar triangles)

= 4/6 = 2/3

Answer: AE/AC = 2/3

6. (a) The surface area of a sphere, 1 mm^2 varies directly as the square of its diameter d mm. If the surface area is to be doubled, what ratio must the diameter be altered?

Surface area $S \propto d^2 \rightarrow S = kd^2$

If S doubles: $2S = k(d')^2$

$2(kd^2) = k(d')^2$

$(d')^2 = 2d^2$

$d'/d = \sqrt{2}$

Ratio = $\sqrt{2} : 1$

Answer: $\sqrt{2} : 1$

(b) If [a x - u; a - u b] write x as a subject of the formula

Assume matrix equation: [a x-u; a-u b] represents a system, but interpret as determinant or solve for x.

If [a x-u; a-u b] = 0 (determinant for singularity):

a(b) - (x-u)(a-u) = 0

$ab - (x-u)(a-u) = 0$

$(x-u)(a-u) = ab$

$x-u = ab / (a-u)$

$x = ab / (a-u) + u$

Answer: $x = ab / (a-u) + u$

7. (a) Express 2/3 as 3 integers in a simplified form

2/3 is already a fraction, not expressible as 3 integers. Possibly a typo; if meant as ratio 2:3, it's 2/3.

Assume ratio 2:3:5 (common form):

2/3 cannot be expressed as 3 integers directly.

Answer: 2/3 (cannot express as 3 integers; possible typo)

(b) The sides of a rectangle are in the ratio 3:5. If the perimeter of this rectangle is 800 cm; find the dimensions of the rectangle

Ratio 3:5 → Let sides be 3x and 5x.

Perimeter = $2(3x + 5x) = 16x$

$16x = 800$

$x = 50$

Dimensions: $3x = 150$ cm, $5x = 250$ cm

Answer: 150 cm, 250 cm

8. (a) If the third term of a geometric progression is 100 and the sixth term is 800, find the fifth term and the sum of the first two terms

Third term: $ar^2 = 100$

Sixth term: $ar^5 = 800$

Divide: $ar^5 / ar^2 = 800 / 100$

$r^3 = 8 \rightarrow r = 2$

$ar^2 = 100 \rightarrow a(2^2) = 100 \rightarrow 4a = 100 \rightarrow a = 25$

Fifth term: $ar^4 = 25 \times 2^4 = 25 \times 16 = 400$

First two terms: $a + ar = 25 + 25 \times 2 = 25 + 50 = 75$

Answer: Fifth term = 400, Sum of first two terms = 75

(b) A small business sells products worth 1,000,000 (Tshs) during its first year. The owner of the business has a target of increasing annual sales by 750,000 (Tshs) each year. Assuming this target is met, find the total sales during the first 10 years of the business operation

Arithmetic sequence: a = 1,000,000, d = 750,000, n = 10

$S_n = (n/2)[2a + (n-1)d]$

$S_{10} = (10/2)[2(1,000,000) + (10-1)(750,000)]$

$= 5[2,000,000 + 9(750,000)]$

$= 5[2,000,000 + 6,750,000]$

$= 5 \times 8,750,000 = 43,750,000$

Answer: 43,750,000 Tshs

9. (a) Given that x is an acute angle and that sin x = 2/5, find the value of tan x

sin x = 2/5, acute angle.

$\cos x = \sqrt{(1 - \sin^2 x)} = \sqrt{(1 - (2/5)^2)} = \sqrt{(1 - 4/25)} = \sqrt{(21/25)} = \sqrt{21}/5$

$\tan x = \sin x / \cos x = (2/5) / (\sqrt{21}/5) = 2 / \sqrt{21}$

Rationalize: $2\sqrt{21} / 21$

Answer: $2\sqrt{21} / 21$

(b) An observer on the top of a cliff, 25 m above sea level, views a boat on the sea at an angle of depression of 60°. How far is the boat from the top of the cliff?

Angle of depression = 60°, height = 25 m.

tan 60° = opposite/adjacent = 25/distance

$\sqrt{3} = 25$ / distance

distance = $25 / \sqrt{3}$

Rationalize: $25\sqrt{3} / 3$

Answer: $25\sqrt{3} / 3$ m

(c) By factorization, find the solution set for $x^2 - x - 6 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x - 3 = 0 \rightarrow x = 3$

$x + 2 = 0 \rightarrow x = -2$

Solution set: {3, -2}

Answer: {3, -2}

(d) Solve the simultaneous equations given below by elimination method

$3x - y = 23$

$4x + 3y = 48$

Multiply first by 3: $9x - 3y = 69$

Add to second: $9x - 3y + 4x + 3y = 69 + 48$

$13x = 117$

$x = 9$

Substitute $x = 9$ into $3x - y = 23$:

$3(9) - y = 23 \rightarrow 27 - y = 23 \rightarrow y = 4$

Answer: $x = 9$, $y = 4$

SECTION B (40 Marks)

Answer four (4) questions from this section. Extra question will not be marked

11. (a) Find the greatest value of the function $f(x, y) = 7x + 3y$ subject to the constraints:

$2x + 3y \leq 12$

$x + 3y \geq 9$

$x \geq 0, y \geq 0$

Vertices:

(0, 3): $2x + 3y = 12 \rightarrow y = 4$, $x + 3y \geq 9 \rightarrow y \geq 3$

(0, 4): $2x + 3y = 12 \rightarrow y = 4$

(6, 0): $2x + 3y = 12 \rightarrow x = 6$

(3, 1): $2x + 3y = 12 \rightarrow 6 + 3y = 12 \rightarrow y = 2$; $x + 3y = 9 \rightarrow x + 3 = 9 \rightarrow x = 3$

$f(x, y) = 7x + 3y$:

(0, 3): $7(0) + 3(3) = 9$

(0, 4): $7(0) + 3(4) = 12$

(6, 0): $7(6) + 3(0) = 42$

(3, 1): $7(3) + 3(1) = 21 + 3 = 24$

Greatest value = 42 at (6, 0).

Answer: Greatest value = 42 at x = 6, y = 0

(b) The curve $y = ax^2 + bx + c$ passes through the points (1, 8), (0, 5) and (3, 20). Find the values of a, b and c and hence the equation of the curve

(0, 5): $c = 5$

(1, 8): $a(1)^2 + b(1) + 5 = 8 \rightarrow a + b + 5 = 8 \rightarrow a + b = 3$

(3, 20): $a(3)^2 + b(3) + 5 = 20 \rightarrow 9a + 3b + 5 = 20 \rightarrow 9a + 3b = 15 \rightarrow 3a + b = 5$

Solve: $3a + b = 5$, $a + b = 3$

Subtract: $(3a + b) - (a + b) = 5 - 3$

$2a = 2 \rightarrow a = 1$

$a + b = 3 \rightarrow 1 + b = 3 \rightarrow b = 2$

$c = 5$

Equation: $y = x^2 + 2x + 5$

Answer: a = 1, b = 2, c = 5; $y = x^2 + 2x + 5$

12. Carefully study the frequency distribution table which shows marks for 40 students in History examination

Marks	1 - 20	21 - 40	41 - 60	61 - 80	81 - 100
Number of students	3	10	12	8	7

(a) Determine the mean, given the assumed mean is 50

Class	Midpoint	d = (x-50)/20	f	fd

1-20 | 10.5 | (10.5-50)/20 = -1.975 | 3 | -5.925

21-40 | 30.5 | (30.5-50)/20 = -0.975 | 10 | -9.75

41-60 | 50.5 | (50.5-50)/20 = 0.025 | 12 | 0.3

61-80 | 70.5 | (70.5-50)/20 = 1.025 | 8 | 8.2

81-100 | 90.5 | (90.5-50)/20 = 2.025 | 7 | 14.175

$\Sigma f = 40$, $\Sigma fd = 7$

Mean = 50 + (7/40) × 20 = 50 + 3.5 = 53.5

Answer: Mean = 53.5

(b) the median

Cumulative frequency:

1-20: 3

21-40: 13

41-60: 25

61-80: 33

81-100: 40

Median position = (40+1)/2 = 20.5th, in 41-60 class

Median = L + [(n/2 - cf)/f] × c

L = 41, n/2 = 20, cf = 13, f = 12, c = 20

Median = 41 + [(20 - 13)/12] × 20 = 41 + (7/12) × 20 = 41 + 11.67 = 52.67

Answer: Median = 52.67

(c) Modal class and its corresponding class mark

Modal class: 41-60 (frequency = 12)

Class mark = (41 + 60) / 2 = 50.5

Answer: Modal class = 41-60, Class mark = 50.5

13. The figure below shows a rectangular prism in which AB = 16 cm, BC = 12 cm and QC = 5 cm

Calculate

(a) its total surface area

Dimensions: AB = 16 cm, BC = 12 cm, QC = 5 cm

Surface area = $2(16\times12 + 16\times5 + 12\times5)$

$= 2(192 + 80 + 60) = 2(332) = 664$ cm²

Answer: 664 cm²

(b) the angle between PB and the plane ABCD

PB = $\sqrt{(16^2 + 12^2 + 5^2)} = \sqrt{(256 + 144 + 25)} = \sqrt{425}$

Plane ABCD normal = (0, 0, 1), PB vector = (16, 12, 5)

Projection of PB on ABCD = $\sqrt{(16^2 + 12^2)} = \sqrt{400} = 20$

$\cos\theta = 5 / \sqrt{425}$

$\theta = \cos^{-1}(5/\sqrt{425}) \approx 76.1°$

Answer: 76.1°

(c) the volume in litres the prism can hold (1 litre = 1000 cm³)

Volume = $16 \times 12 \times 5 = 960$ cm³

In litres: 960 / 1000 = 0.96 litres

Answer: 0.96 litres

.

14. The following information relates to Mr. Kazimoto, a trader, as at 30th July 2004:

Sales shs. 340,000.00

Cost of sales 75% of sales

Opening stock shs. 90,000.00

Net profit 20% of cost of goods sold

Closing stock 20% of cost of goods sold

Calculate:

(a) Purchases

Sales = 340,000

Cost of sales = 75% of sales = 0.75 × 340,000 = 255,000

Cost of sales = Opening stock + Purchases - Closing stock

Closing stock = 20% of cost of goods sold = 0.20 × 255,000 = 51,000

255,000 = 90,000 + Purchases - 51,000

255,000 = 39,000 + Purchases

Purchases = 255,000 - 39,000 = 216,000

Answer: 216,000 shs

(b) Cost of sales

Already calculated: Cost of sales = 75% of sales = 255,000

Answer: 255,000 shs

(c) Closing stock

Closing stock = 20% of cost of goods sold = 51,000 (from part a)

Answer: 51,000 shs

(d) Net profit

Net profit = 20% of cost of goods sold = 0.20 × 255,000 = 51,000

Answer: 51,000 shs

(e) Expenses

Net profit = Sales - Cost of sales - Expenses

51,000 = 340,000 - 255,000 - Expenses

51,000 = 85,000 - Expenses

Expenses = 85,000 - 51,000 = 34,000

Answer: 34,000 shs

15. (a) A translation takes the point (8, 5) to (12, -4). Find where it will take the point (5, 4)

Translation vector: (12 - 8, -4 - 5) = (4, -9)

Apply to (5, 4): (5 + 4, 4 - 9) = (9, -5)

Answer: (9, -5)

(b) A linear transformation T maps (x, y) onto (x', y') such that

[x'] = [2 -4] [x] + [8]

[y'] = [-1 3] [y] + [-4]

Find the image of (2, -3) under T

T(x, y) = [2 -4; -1 3] [x; y] + [8; -4]

[2 -4; -1 3] [2; -3] = [2(2) + (-4)(-3); (-1)(2) + 3(-3)] = [4 + 12; -2 - 9] = [16; -11]

Add translation: [16 + 8; -11 - 4] = [24; -15]

Answer: (24, -15)

(c) A point (x, y) is reflected on the line y = x followed by a rotation through an angle of 180° clockwise about the origin. Find the image of (2, 3) under this double transformation

Reflection over y = x: (x, y) → (y, x) → (2, 3) → (3, 2)

Rotation 180° clockwise: (x, y) → (-x, -y) → (3, 2) → (-3, -2)

Answer: (-3, -2)

16. (a) IF $f(x) = x^2 - 4x + 3$

Find

(i) $f^{-1}(x)$

$f(x) = x^2 - 4x + 3$

$y = x^2 - 4x + 3$

Complete the square: $x^2 - 4x = (x - 2)^2 - 4$

$y = (x - 2)^2 - 4 + 3 = (x - 2)^2 - 1$

$y + 1 = (x - 2)^2$

$x - 2 = \pm\sqrt{(y + 1)}$

$x = 2 \pm \sqrt{(y + 1)}$

$f^{-1}(x) = 2 \pm \sqrt{(x + 1)}$ (not a function unless domain restricted; typically $f(x) \geq -1$, so take positive root for inverse)

$f^{-1}(x) = 2 + \sqrt{(x + 1)}$ (for $f(x) \geq -1$)

Answer: $f^{-1}(x) = 2 + \sqrt{(x + 1)}$

(ii) the domain and range of f(x)

$f(x) = x^2 - 4x + 3 = (x - 2)^2 - 1$

Minimum at $x = 2$: $f(2) = -1$

Domain: All real numbers, $(-\infty, \infty)$

Range: $f(x) \geq -1$, $[-1, \infty)$

Answer: Domain: $(-\infty, \infty)$, Range: $[-1, \infty)$

(b) IF the probability that Ali will pass Mathematics is 0.3 and the probability that he will pass Biology is 0.6, find the probability that:

(i) He will pass both subjects

P(Math and Bio) = P(Math) × P(Bio) (independent)

$= 0.3 \times 0.6 = 0.18$

Answer: 0.18

(ii) He will fail both subjects

P(fail Math) = 1 - 0.3 = 0.7

P(fail Bio) = 1 - 0.6 = 0.4

P(fail both) = 0.7 × 0.4 = 0.28

(c) IF A is the event 'Ali will pass Mathematics' and B is the event 'Ali will pass Biology' show whether or not A and B are independent events. [use the information given in part (b) above]

A and B independent if $P(A \cap B) = P(A) \times P(B)$

$P(A \cap B) = 0.18$ (from part b(i))

$P(A) \times P(B) = 0.3 \times 0.6 = 0.18$

Since $P(A \cap B) = P(A) \times P(B)$, A and B are independent.