THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2010**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) Write 624.3278 correct to:

(i) five (5) significant figures

624.3278 has significant digits 6, 2, 4, 3, 2, 7, 8.

To five significant figures: 6, 2, 4, 3, 2. The next digit is $7 \geq 5$, so round up:

$2 \rightarrow 3$

Answer: 624.33

(ii) three (3) decimal places

624.3278 to three decimal places:

The third decimal place is 3, the fourth is $2 < 5$, so no rounding up.

Answer: 624.328

(b) A mathematics teacher bought 40 expensive calculators at shs.16,400 each and a number of other cheaper calculators costing shs.5,900 each. She spent a total of shs. 774,000. How many of the cheaper calculators did she buy?

Cost of expensive calculators: $40 \times 16{,}400 = 656{,}000$

Total spent: 774,000

Cost of cheaper calculators: $774{,}000 - 656{,}000 = 118{,}000$

Number of cheaper calculators: $118{,}000 / 5{,}900 = 20$

Answer: 20

2. (a) EVALUATE without using mathematical tables $2 \log 5 + \log 36 - \log 9$

Use logarithm properties: $\log a + \log b = \log(ab)$, $\log a - \log b = \log(a/b)$.

$2 \log 5 + \log 36 - \log 9 = \log (5^2) + \log 36 - \log 9$

$= \log 25 + \log 36 - \log 9$

$= \log (25 \times 36 / 9)$

$= \log (25 \times 4)$

$= \log 100$

$= 2$ (since $\log 100 = 2$, assuming base 10)

Answer: 2

(b) Simplify $72^{2+3} \times 6^{3+-3} / 3^{9+-3}$

Rewrite with positive exponents:

$72 = 72^1$, so $72^{2+3} = 72^5$

$6^{3+-3} = 6^{3-3} = 6^0 = 1$

$3^{9+-3} = 3^{9-3} = 3^6$

Expression: $(72^5 \times 6^0) / 3^6 = 72^5 / 3^6$

$72 = 8 \times 9 = 2^3 \times 3^2$, so $72^5 = (2^3 \times 3^2)^5 = 2^{15} \times 3^{10}$

$3^6 = 3^6$

$72^5 / 3^6 = (2^{15} \times 3^{10}) / 3^6 = 2^{15} \times 3^{10-6} = 2^{15} \times 3^4$

Answer: $2^{15} \times 3^4$

3. (a) GIVEN that $A = \{x : 0 \leq x \leq 8\}$ $B = \{1, 3, 5, 7, 11\}$ where x is an integer, in the same form, represent in a Venn diagram

(i) $A \cap B$

$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{1, 3, 5, 7, 11\}$

$A \cap B = \{1, 3, 5, 7\}$ (elements in both A and B)

Answer: $A \cap B = \{1, 3, 5, 7\}$

(ii) $A \cap B$

This likely means $A \cup B$ (union, as $\cap$ is already asked).

$A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 11\}$ (all elements in A or B)

Answer: $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 11\}$

(b) In a sch. of 75 pupils, 43% of the pupils take Biology but not Chemistry, 32% pupils take Chemistry but not Biology. How many pupils do not take either Biology or Chemistry?

Total pupils = 75

Biology but not Chemistry: 43% of 75 = $0.43 \times 75 = 32.25$, round to 32

Chemistry but not Biology: 32% of 75 = $0.32 \times 75 = 24$

Pupils taking at least one subject = 32 + 24 = 56

Pupils taking neither = 75 - 56 = 19

Answer: 19

4. (a) Without using mathematical tables, find the numerical value of

(i) $\sin 45° + \cos 45° \tan 45°$

$\sin 45° = \sqrt{2}/2$, $\cos 45° = \sqrt{2}/2$, $\tan 45° = 1$

$\sin 45° + \cos 45° \tan 45° = \sqrt{2}/2 + (\sqrt{2}/2)(1)$

$= \sqrt{2}/2 + \sqrt{2}/2 = 2(\sqrt{2}/2) = \sqrt{2}$

Answer: $\sqrt{2}$

(ii) Write down the equation of the line which passes through (7, 3) and which is inclined at 45° to the positive direction of the x-axis

Slope at 45°: $\tan 45° = 1$

Equation: $y - y_1 = m(x - x_1)$

$y - 3 = 1(x - 7)$

$y - 3 = x - 7$

$y = x - 4$

Answer: $y = x - 4$

(b) The position vectors of the points A, B and C are 4i + 3j, [4i + 3j], [-5i + j] respectively. Find the vectors AB, BC and AC hence verify that AB + BC = AC

A = 4i + 3j, B = 4i + 3j (same as A, possible error), C = -5i + j

AB = B - A = (4i + 3j) - (4i + 3j) = 0 (if B = A, let's assume B is different, e.g., 4i + 3j was repeated; need correct B)

Assume B = 4i + 3j, try next point:

BC = C - B = (-5i + j) - (4i + 3j) = -9i - 2j

AC = C - A = (-5i + j) - (4i + 3j) = -9i - 2j

AB + BC = 0 + (-9i - 2j) = -9i - 2j = AC

Since $AB = 0$, this suggests A and B are the same point, which may be a typo. If B is distinct, the verification holds similarly.

Answer: Verification depends on correct B; if distinct, $AB + BC = AC$ holds.

5. (a) The volume of two similar cylinders is 125 cm^3 and 512 cm^3. If the radius of the larger cylinder is 8cm, find the radius of the smaller cylinder

Volume ratio: $125 / 512$

For similar cylinders, volume ratio = $(\text{scale factor})^3$

$(125 / 512) = (r_1/r_2)^3$

$r_2 = 8$ cm (larger), r_1 = smaller

$(r_1/8)^3 = 125 / 512$

$r_1/8 = (125 / 512)^{(1/3)} = ((5/8)^3)^{(1/3)} = 5/8$

$r_1 = 8 \times 5/8 = 5$

Answer: 5 cm

(b) In the diagram below, show that $AD/CD = AB/AC$

Triangle ABC, D on AB, $CD \perp AB$.

In triangles ADC and ABC:

Angle ADC = Angle ABC (both 90° due to $CD \perp AB$)

Angle DAC = Angle BAC (common)

Triangles are similar (AA similarity).

$AD/AB = CD/AC$ (corresponding sides)

Cross-multiply: $AD \times AC = AB \times CD$

Divide by $CD \times AC$: $AD/CD = AB/AC$

Answer: $AD/CD = AB/AC$ (shown)

6. (a) Juma bought motor vehicle spare parts from Japan worth 5,900,000 Japanese Yen. When he arrived in Tanzania he was charged custom duty of 25% on the spare parts. If the exchange rates were as follows: 1 US dollar = 118 Japanese Yen 1 US dollar = 1,250 Tanzania shillings. Calculate the duty he paid in Tanzania shillings

Cost in Yen: 5,900,000

Yen to USD: $5,900,000 / 118 = 50,000$ USD

USD to TZS: $50,000 \times 1,250 = 62,500,000$ TZS

Duty = 25% of $62,500,000 = 0.25 \times 62,500,000 = 15,625,000$ TZS

Answer: 15,625,000 TZS

(b) The distance of the horizon of 1km varies as the square root of the height h m of the observer above sea level. An observer at a height of 100m above sea level sees the horizon at a distance of 35.7 km

(i) Find the distance of the horizon from an observer 70m above sea level

$d \propto \sqrt{h} \rightarrow d = k\sqrt{h}$

At $h = 100$, $d = 35.7$: $35.7 = k\sqrt{100} \rightarrow 35.7 = k(10) \rightarrow k = 3.57$

At $h = 70$: $d = 3.57\sqrt{70} \approx 3.57 \times 8.367 = 29.87$

Answer: 29.87 km

(ii) Find an equation connecting d and h

$d = 3.57\sqrt{h}$

Answer: $d = 3.57\sqrt{h}$

7. (a) An amount of Tshs. 12,000 is to be shared among Ali, Anna and Juma in the ratio 2:3:5 respectively. How much will each get?

Total parts = $2 + 3 + 5 = 10$

1 part = $12,000 / 10 = 1,200$

Ali: $2 \times 1,200 = 2,400$

Anna: $3 \times 1,200 = 3,600$

Juma: $5 \times 1,200 = 6,000$

Answer: Ali: 2,400, Anna: 3,600, Juma: 6,000 TZS

(b) A certain worker used his salary as follows: 20% on house rent, 45% on food, 10% on refreshment and 15% on school fees. If he/she was left with Tshs 22,000, determine:

(i) The salary of this worker

Let salary = S.

Total spent = 20% + 45% + 10% + 15% = 90%

Remaining = 10% of S = 22,000

0.10S = 22,000

S = 22,000 / 0.10 = 220,000

Answer: 220,000 TZS

(ii) The amount of money which he/she spent on food

Food = 45% of 220,000 = 0.45 × 220,000 = 99,000

Answer: 99,000 TZS

8. (a) Find the general term and hence the 30^{th} term of the sequence 1, 2, 4, 8, …

Geometric sequence: first term a = 1, common ratio r = 2

General term: $T_n = ar^{n-1} = 1 \times 2^{n-1} = 2^{n-1}$

30th term: $T_{30} = 2^{30-1} = 2^{29}$

Answer: General term: 2^{n-1}, 30th term: 2^{29}

(b) Given the series 100 + 92 + 84 + …

(i) Find the 20^{th} term

Arithmetic sequence: a = 100, d = 92 - 100 = -8

$T_n = a + (n-1)d$

$T_{20} = 100 + (20-1)(-8) = 100 + 19(-8) = 100 - 152 = -52$

Answer: -52

(ii) the sum of the first 20 terms

$S_n = (n/2)[2a + (n-1)d]$

$S_{20} = (20/2)[2(100) + (20-1)(-8)]$

= 10[200 + 19(-8)]

= 10[200 - 152]

= 10 × 48 = 480

Answer: 480

9. (a) IF $\tan A = 3/4$ and A is acute, find $\cos A$, $\sin A$ and hence verify the identity $\cos A + \sin^2 A = 1$

$\tan A = 3/4$, so opposite = 3, adjacent = 4.

Hypotenuse = $\sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$

$\cos A$ = adjacent/hypotenuse = 4/5

$\sin A$ = opposite/hypotenuse = 3/5

Verify: $\cos A + \sin^2 A = 4/5 + (3/5)^2 = 4/5 + 9/25 = 20/25 + 9/25 = 29/25 \neq 1$

The identity given ($\cos A + \sin^2 A = 1$) does not hold; it should likely be $\sin^2 A + \cos^2 A = 1$:

$\sin^2 A + \cos^2 A = (3/5)^2 + (4/5)^2 = 9/25 + 16/25 = 25/25 = 1$, which holds.

Answer: $\cos A = 4/5$, $\sin A = 3/5$, $\sin^2 A + \cos^2 A = 1$ verified.

(b) Given the right angled triangle above whose sides are measured in centimeter determine:

(i) the value of x

The triangle has sides: opposite = $2x + 1$, adjacent = $x - 1$, hypotenuse = $2x$.

Pythagorean theorem: $(2x + 1)^2 + (x - 1)^2 = (2x)^2$

$(2x + 1)^2 = 4x^2 + 4x + 1$

$(x - 1)^2 = x^2 - 2x + 1$

$(2x)^2 = 4x^2$

$4x^2 + 4x + 1 + x^2 - 2x + 1 = 4x^2$

$5x^2 + 2x + 2 = 4x^2$

$x^2 + 2x + 2 = 0$

Discriminant = $2^2 - 4(1)(2) = 4 - 8 = -4 < 0$, no real solutions.

Answer: No real solution

(ii) the area of the triangle

Since x has no real solution, area cannot be computed without correct x. If we assume a corrected triangle, we'd need valid sides.

10. (a) Factorize each of the following expressions:

(i) $3c^2 - 5c^3d - 3b^2c + 5b^3d$

Group terms: $(3c^2 - 5c^3d) + (-3b^2c + 5b^3d)$

$= c(3c - 5c^2d) + b(-3b^2 + 5b^2d)$

$= c(3 - 5cd) - b(3b^2 - 5b^2d)$

This grouping doesn't factor easily. Try pairing differently:

$(3c^2 - 3b^2c) + (-5c^3d + 5b^3d)$

$= 3c(c - b^2) + 5d(-c^3 + b^3)$

$= 3c(c - b^2) + 5d(b^3 - c^3)$

$= 3c(c - b^2) + 5d(b - c)(b^2 + bc + c^2)$

No common factors. The expression may not factor simply.

Answer: Cannot factor further with simple integers.

(ii) $3(2 - y^2) - 17y$ which satisfies the equation $3(2 - y^2) - 17y = 0$

Factor: $3(2 - y^2) - 17y = 6 - 3y^2 - 17y$

$= -3y^2 - 17y + 6$

Solve quadratic: $3y^2 + 17y - 6 = 0$

$3y^2 + 18y - y - 6 = 0$

$3y(y + 6) - 1(y + 6) = 0$

$(3y - 1)(y + 6) = 0$

$y = 1/3$ or $y = -6$

The expression factors as $-1(3y - 1)(y + 6)$.

Answer: $-1(3y - 1)(y + 6)$

SECTION B (40 Marks)

Answer four (4) questions from this section. Extra questions will not be marked

11. (a) Maximize $f = 2y - x$ subject to the following constraints:

$x \geq 0$

$y \geq 0$

$2x + y \leq 6$

$x + 2y \leq 6$

Vertices of the feasible region:

(0, 0): $x = 0$, $y = 0$

(0, 3): $2x + y = 6 \rightarrow y = 6$; $x + 2y = 6 \rightarrow 2y = 6 \rightarrow y = 3$

(3, 0): $2x + y = 6 \rightarrow 2x = 6 \rightarrow x = 3$; $x + 2y = 6 \rightarrow x = 3$

Intersection of $2x + y = 6$ and $x + 2y = 6$:

$2x + y = 6$

$x + 2y = 6$

Multiply second by 2: $2x + 4y = 12$

Subtract: $(2x + 4y) - (2x + y) = 12 - 6$

$3y = 6 \rightarrow y = 2$

$x + 2(2) = 6 \rightarrow x + 4 = 6 \rightarrow x = 2$

Vertex: (2, 2)

Evaluate $f = 2y - x$:

(0, 0): $f = 0$

(0, 3): $f = 2(3) - 0 = 6$

(3, 0): $f = 0 - 3 = -3$

(2, 2): $f = 2(2) - 2 = 2$

Maximum $f = 6$ at (0, 3).

Answer: Maximum $f = 6$ at $x = 0$, $y = 3$

(b) Sara had 300 shillings to buy erasers and pencils. An eraser costs 20 shillings while a pencil costs 30 shillings. IF the number of erasers bought is at least twice the number of pencils, formulate the inequalities that represent this information

Let x = number of erasers, y = number of pencils.

Cost: $20x + 30y \leq 300 \rightarrow 2x + 3y \leq 30$

Erasers at least twice pencils: $x \geq 2y$

Non-negative: $x \geq 0$, $y \geq 0$

Answer: $2x + 3y \leq 30$, $x \geq 2y$, $x \geq 0$, $y \geq 0$

12. The data below represent masses in kg of 36 men

51 61 60 70 75 71 75 70 74 73 72 82

70 71 76 74 50 68 65 69 65 55 72 69

64 83 63 74 80 90 59 65 62 64 61 66

(i) Prepare a frequency distribution table of class interval of size 5 beginning with the number 50 taking into consideration that both lower limits and upper class limits are inclusive

Classes: 50-54, 55-59, …, 90-94

50-54: 51, 50 → 2

55-59: 55, 59 → 2

60-64: 61, 60, 62, 64, 61, 63, 64 → 7

65-69: 65, 69, 65, 69, 66, 68, 65 → 7

70-74: 70, 71, 70, 74, 73, 72, 70, 74, 72 → 9

75-79: 75, 71, 75, 76, 74 → 5

80-84: 80, 82, 83 → 3

85-89: 0

90-94: 90 → 1

Frequency table:

Class	Frequency
50-54	2
55-59	2
60-64	7

65-69	7
70-74	9
75-79	5
80-84	3
85-89	0
90-94	1

(ii) Calculate the mean and mode from the frequency distribution table prepared in (i) above by using assumed mean from the class mark of the modal class

Modal class: 70-74 (frequency = 9)

Class mark = (70 + 74) / 2 = 72 (assumed mean A)

Class , Midpoint , d = (x - A)/5 , f , fd

50-54 , 52 , (52-72)/5 = -4 , 2 , -8

55-59 , 57 , (57-72)/5 = -3 , 2 , -6

60-64 , 62 , -2 , 7 , -14

65-69 , 67 , -1 , 7 , -7

70-74 , 72 , 0 , 9 , 0

75-79 , 77 , 1 , 5 , 5

80-84 , 82 , 2 , 3 , 6

85-89 , 87 , 3 , 0 , 0

90-94 , 92 , 4 , 1 , 4

, , , Total f = 36, Σfd = -20

Mean = A + (Σfd / Σf) × class width = 72 + (-20 / 36) × 5 = 72 - 100/36 = 72 - 2.78 = 69.22

Mode = $L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times c$

$L = 70, f_1 = 9, f_0 = 7, f_2 = 5, c = 5$

Mode = 70 + [(9 - 7) / (2(9) - 7 - 5)] × 5 = 70 + (2 / 6) × 5 = 70 + 1.67 = 71.67

Answer: Mean = 69.22, Mode = 71.67

13. (a) Below is a circle with centre O and radius r units. By considering the circumference of the circle, the area of the circle, the given angle 0 and the degree measures of a circle (360°), develop the formula for finding

(i) arc length AB

Circumference = $2\pi r$

Angle θ corresponds to arc AB.

Arc length AB = $(\theta / 360) \times 2\pi r$

Answer: Arc length AB = $(\theta / 360) \times 2\pi r$

(ii) area of sector AOB

Area of circle = πr^2

Sector area AOB = $(\theta / 360) \times \pi r^2$

Answer: Area of sector AOB = $(\theta / 360) \times \pi r^2$

(b) Find

(i) the length of arc AB

$\theta = 57°$, $r = 5.4$ cm, $\pi = 22/7$

Arc length = $(\theta / 360) \times 2\pi r$

$= (57 / 360) \times 2 \times (22/7) \times 5.4$

$= (57 / 360) \times (44/7) \times 5.4$

$= (57 \times 44 \times 5.4) / (360 \times 7)$

$= 13543.2 / 2520 \approx 5.37$ cm

Answer: 5.37 cm

(ii) the area of the sector AOB

Area = $(\theta / 360) \times \pi r^2$

$= (57 / 360) \times (22/7) \times (5.4)^2$

= (57 / 360) × (22/7) × 29.16

= (57 × 22 × 29.16) / (360 × 7)

= 36581.52 / 2520 ≈ 14.52 cm²

Answer: 14.52 cm²

14. From 1th January to 29th January 2006 Mr. Bin decided to keep records of his business as follows:

Jan. 1 Mr. Bin started a business with capital in cash 500,000.00

5 Purchased goods 254,000.00

6 Sold goods 290,000.00

9 Purchased goods 204,000.00

10 Expenses 24,000.00

29 Sold goods 320,000.00

YOU ARE REQUIRED TO:

(a) prepare the trial balance

Trial Balance as of 29th January 2006

Account	Debit (Tshs)	Credit (Tshs)
Capital		500000
Cash	308000	
Purchases	458000	
Sales		610000
Expenses	24000	
Total	790000	1110000

Cash calculation: 500,000 - 254,000 + 290,000 - 204,000 - 24,000 + 320,000 = 308,000

Note: Trial balance does not balance; discrepancy suggests missing entries (e.g., closing stock).

(b) open capital and cash account

Capital Account

Details | Amount (Tshs) | Details | Amount (Tshs)

| | Balance b/d | 500,000

Total | 0 | Total | 500,000

Cash Account

Details | Amount (Tshs) | Details | Amount (Tshs)

Capital | 500,000 | Purchases | 254,000

Sales | 290,000 | Purchases | 204,000

Sales | 320,000 | Expenses | 24,000

| | Balance c/d | 308,000

Total | 1,110,000 | Total | 1,110,000

Answer: Capital: 500,000 (credit), Cash: 308,000 (debit)

N.B ALL payments and receipts were made in cash

15. (a) A transformation T has the matrix T = [1 x; r -2]. Under the same transformation T, the point (-4, 1) is mapped onto the point (6, 3). Find x and r

T = [1 x; r -2], maps (-4, 1) to (6, 3).

[1 x; r -2] [-4; 1] = [6; 3]

$1(-4) + x(1) = 6 \rightarrow -4 + x = 6 \rightarrow x = 10$

$r(-4) + (-2)(1) = 3 \rightarrow -4r - 2 = 3 \rightarrow -4r = 5 \rightarrow r = -5/4$

Answer: x = 10, r = -5/4

(b) For what values of n will the matrix [n-1 n+3; 1 6n] be non-singular?

Matrix = [n-1 n+3; 1 6n]. Non-singular if determinant $\neq 0$.

Determinant = (n-1)(6n) - (n+3)(1)

$= 6n^2 - 6n - (n + 3)$

$= 6n^2 - 6n - n - 3$

$= 6n^2 - 7n - 3$

Set determinant $\neq 0$:

$6n^2 - 7n - 3 \neq 0$

Solve $6n^2 - 7n - 3 = 0$ to find singular points:

$6n^2 - 9n + 2n - 3 = 0$

$3n(2n - 3) + 1(2n - 3) = 0$

$(3n + 1)(2n - 3) = 0$

$n = -1/3$ or $n = 3/2$

Non-singular when $n \neq -1/3$ and $n \neq 3/2$.

Answer: $n \neq -1/3$, $n \neq 3/2$

16. (a) If $f(x) = -2x + 3$ find $f^{-1}(3)$

$f(x) = -2x + 3$

$y = -2x + 3$

$x = -2y + 3$

$2y = 3 - x$

$y = (3 - x) / 2$

$f^{-1}(x) = (3 - x) / 2$

$f^{-1}(3) = (3 - 3) / 2 = 0$

Answer: 0

(b) Draw the graph of $f(x) = |x-1|$ for $-4 \leq x \leq 4$

Compute points for $f(x) = |x-1|$:

$x = -4$: $f(x) = |-4-1| = 5$

$x = -3$: $f(x) = 4$

$x = -2$: $f(x) = 3$

$x = -1$: $f(x) = 2$

$x = 0$: $f(x) = 1$

$x = 1$: $f(x) = 0$

$x = 2$: $f(x) = 1$

$x = 3$: $f(x) = 2$

$x = 4$: $f(x) = 3$

V-shape with vertex at (1, 0).

(c) State the domain and range of $f(x) = |x-1|$

Domain: Given as $-4 \leq x \leq 4$

Range: Minimum $f(x) = 0$ at $x = 1$, maximum $f(x) = 5$ at $x = -4$

Range: [0, 5]

Answer: Domain: $-4 \leq x \leq 4$, Range: [0, 5]

(d) The probability that Rose and Juma will be selected for A - level studies after completing their O - level studies are 0.4 and 0.7 respectively. Calculate the probability that:

(i) both of them will be selected

P(Rose and Juma) = P(Rose) × P(Juma) (independent events)

$= 0.4 \times 0.7 = 0.28$

Answer: 0.28

(ii) either Rose or Juma will be selected

P(Rose or Juma) = P(Rose) + P(Juma) - P(Rose and Juma)

$= 0.4 + 0.7 - 0.28 = 0.82$

Answer: 0.82