THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2011**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) Express 0.5473

(i) correct to three (3) significant figures

0.5473 to three significant figures:

The first three significant figures are 5, 4, and 7. The next digit is 3, which is less than 5, so we do not round up.

Answer: 0.547

(ii) correct to three (3) decimal places

0.5473 to three decimal places:

The number is already at four decimal places. The third decimal place is 7, and the fourth decimal place is 3, which is less than 5, so we do not round up.

Answer: 0.547

(b) EVALUATE $0.0944 \times 1.21 \div 5.36^{-3}$ without using mathematical tables and express the answer as a fraction in its simplest form

First, calculate 5.36^{-3}:

$5.36^{-3} = 1 / (5.36)^3$

$5.36^2 = 5.36 \times 5.36 = 28.7296$

$(5.36)^3 = 28.7296 \times 5.36 = 154.000896$

$5.36^{-3} = 1 / 154.000896 \approx 0.00649348$

Now compute 0.0944×1.21:

$0.0944 \times 1.21 = 0.114224$

Now multiply by 5.36^{-3}:

$0.114224 \times 0.00649348 \approx 0.000741664$

To express as a fraction:

$0.0944 = 944/10000 = 236/2500$

$1.21 = 121/100$

$5.36 = 536/100 = 134/25$

$(5.36)^3 = (134/25)^3 = 134^3 / 25^3$

$134^2 = 17956$

$134^3 = 17956 \times 134 = 2406104$

$25^3 = 15625$

$(5.36)^3 = 2406104 / 15625$

$5.36^{-3} = 15625 / 2406104$

So, $0.0944 \times 1.21 \div 5.36^{-3} = (944/10000) \times (121/100) \times (15625 / 2406104)$

$= (944 \times 121 \times 15625) / (10000 \times 100 \times 2406104)$

Numerator: $944 \times 121 = 114224$

$114224 \times 15625 = 1784750000$

Denominator: $10000 \times 100 \times 2406104 = 2406104000000$

Fraction: 1784750000 / 2406104000000

Simplify by dividing by 1000:

1784750 / 2406104000

Divide by 250:

7147 / 9624416

Since 7147 is prime and does not divide 9624416, this is the simplest form.

Answer: 7147 / 9624416

2. (a) Solve the equation $\log_5 x + \log_5(x + 2) - \log_5 3 = 0$

Use the property of logarithms: $\log_5 a + \log_5 b = \log_5(ab)$ and $\log_5 a - \log_5 b = \log_5(a/b)$:

$\log_5 x + \log_5(x + 2) - \log_5 3 = \log_5(x(x + 2) / 3)$

$\log_5(x(x + 2) / 3) = 0$

$\log_5(x(x + 2) / 3) = \log_5 1$

$x(x + 2) / 3 = 1$

$x(x + 2) = 3$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

Since logarithms are defined only for positive values, discard $x = -3$ (as $\log_5(-3)$ is undefined).

Check $x = 1$:

$\log_5 1 + \log_5(1 + 2) - \log_5 3 = \log_5 1 + \log_5 3 - \log_5 3 = 0$, which holds.

Answer: $x = 1$

(b) By rationalizing the denominator, simplify the following expression: $(5\sqrt{3} + \sqrt{7}) / \sqrt{3}$

Multiply numerator and denominator by $\sqrt{3}$:

$(5\sqrt{3} + \sqrt{7}) / \sqrt{3} \times (\sqrt{3} / \sqrt{3}) = (5\sqrt{3} \times \sqrt{3} + \sqrt{7} \times \sqrt{3}) / (\sqrt{3} \times \sqrt{3})$

$= (5 \times 3 + \sqrt{21}) / 3$

$= (15 + \sqrt{21}) / 3$

Answer: $(15 + \sqrt{21}) / 3$

3. (a) A shopkeeper sold 500 sweets. Some cost shs. 5 and some cost shs. 8. The cash received for the more expensive sweets was shs. 100 more than for the cheaper sweets. Find the number of each kind of sweet which were sold

Let x be the number of sweets sold at shs. 5, and y be the number sold at shs. 8.

Total sweets: $x + y = 500$

Cash from shs. 5 sweets: $5x$

Cash from shs. 8 sweets: $8y$

Cash from more expensive sweets is shs. 100 more: $8y = 5x + 100$

Solve:

$x + y = 500 \rightarrow y = 500 - x$

$8y = 5x + 100$

$8(500 - x) = 5x + 100$

$4000 - 8x = 5x + 100$

$4000 - 100 = 5x + 8x$

$3900 = 13x$

$x = 3900 / 13 = 300$

$y = 500 - 300 = 200$

Answer: 300 sweets at shs. 5, 200 sweets at shs. 8

(b) A survey of 240 houses showed that all of them kept a farm or a garden or both. IF 180 kept gardens and 79 kept farms, how many houses kept both?

Total houses = 240

Houses with gardens = 180

Houses with farms = 79

Let z be the number of houses that kept both. Using the inclusion-exclusion principle:

Houses with farms or gardens = Houses with farms + Houses with gardens - Houses with both

$240 = 79 + 180 - z$

$240 = 259 - z$

$z = 259 - 240 = 19$

Answer: 19 houses kept both a farm and a garden.

4. (a) Find the distance between point (-3, -2) and the point midway between (2, 13) and (4, 7). Write your answer in the form $a\sqrt{c}$ where a and c are positive real numbers

Midpoint of (2, 13) and (4, 7):

x-coordinate = $(2 + 4) / 2 = 3$

y-coordinate = $(13 + 7) / 2 = 10$

Midpoint = (3, 10)

Distance between (-3, -2) and (3, 10):

Distance = $\sqrt{[(3 - (-3))^2 + (10 - (-2))^2]}$

$= \sqrt{[(3 + 3)^2 + (10 + 2)^2]}$

$= \sqrt{[6^2 + 12^2]}$

$= \sqrt{[36 + 144]}$

$= \sqrt{180}$

$= \sqrt{(36 \times 5)}$

$= 6\sqrt{5}$

Answer: $6\sqrt{5}$

(b) Given the vectors x = 3i + 2j, v = 5i - 3j and w = 4i - 2j

(i) Find the resultant vector x + v and its direction

x + v = (3i + 2j) + (5i - 3j) = (3 + 5)i + (2 - 3)j = 8i - j

Magnitude = $\sqrt{(8^2 + (-1)^2)} = \sqrt{(64 + 1)} = \sqrt{65}$

Direction (angle θ with positive x-axis): tan θ = y/x = -1/8

$\theta = \tan^{-1}(-1/8) \approx -7.13°$ (in the fourth quadrant, since x is positive and y is negative).

Answer: Resultant = 8i - j, direction ≈ -7.13° from the positive x-axis.

(ii) Plot the three vectors on the same axes and hence indicate the magnitude of each (do not perform any calculation)

To plot:

x = 3i + 2j: From (0,0) to (3,2)

v = 5i - 3j: From (0,0) to (5,-3)

w = 4i - 2j: From (0,0) to (4,-2)

5. (a) In figure 1, ABCD is a square. IF AR = BR prove that R is the midpoint of DC

Given ABCD is a square, let the side length be s. Place the square on a coordinate plane:

A(0,0), B(s,0), C(s,s), D(0,s).

Let R be on DC, so R = (x,s), where $0 \leq x \leq s$.

AR = distance from A(0,0) to R(x,s) = $\sqrt{(x^2 + s^2)}$

BR = distance from B(s,0) to R(x,s) = $\sqrt{((x - s)^2 + s^2)}$

Since AR = BR:

$\sqrt{(x^2 + s^2)} = \sqrt{((x - s)^2 + s^2)}$

Square both sides:

$x^2 + s^2 = (x - s)^2 + s^2$

$x^2 + s^2 = x^2 - 2sx + s^2 + s^2$

$x^2 + s^2 = x^2 - 2sx + 2s^2$

$s^2 = -2sx + 2s^2$

$0 = -2sx + s^2$

$2sx = s^2$

$x = s/2$ (since $s \neq 0$)

R = (s/2, s), which is the midpoint of DC (from (0,s) to (s,s)).

Thus, R is the midpoint of DC.

(b) Calculate the size of an interior angle of a regular nonagon

A regular nonagon has 9 sides.

Interior angle of a regular polygon = $[(n-2) \times 180°] / n$

For n = 9:

Interior angle = $[(9-2) \times 180°] / 9$

$= (7 \times 180) / 9$

$= 1260 / 9$

$= 140°$

Answer: 140°

6. The number of square tiles needed to surface the floor of a hall varies inversely as the square of the length of a side of the tile used. IF 2016 tiles of side 0.4m would be needed to surface the floor of a certain hall, how many tiles of side 0.3m would be required?

The number of tiles N varies inversely as the square of the side length s:

$N = k / s^2$

For s = 0.4 m, N = 2016:

$2016 = k / (0.4)^2$

2016 = k / 0.16

k = 2016 × 0.16 = 322.56

For s = 0.3 m:

N = k / $(0.3)^2$

= 322.56 / 0.09

= 3584

Answer: 3584 tiles

7. The ratio of men : women : children living in Mkuza village is 6 : 7 : 3. IF there are 42,000 women, find how many:

(a) (i) children live in Mkuza village

Ratio men : women : children = 6 : 7 : 3

Women = 7 parts = 42,000

1 part = 42,000 / 7 = 6,000

Children = 3 parts = 3 × 6,000 = 18,000

Answer: 18,000 children

(ii) people altogether live in Mkuza village

Men = 6 parts = 6 × 6,000 = 36,000

Women = 42,000

Children = 18,000

Total people = 36,000 + 42,000 + 18,000 = 96,000

Answer: 96,000 people

(b) The 42,000 women is an increase of 20% on the number of women 10 years ago. How many women lived in the village?

Let the number of women 10 years ago be W.

42,000 = W + 0.20W = 1.20W

W = 42,000 / 1.20

= 35,000

Answer: 35,000 women

8. (a) IF the first term of an arithmetic progression is 3 and the third term is 13, find the second term, the fourth term and the sum of the first ten terms

First term a = 3, third term a + 2d = 13

3 + 2d = 13

2d = 10

d = 5

Second term = a + d = 3 + 5 = 8

Fourth term = a + 3d = 3 + 3(5) = 3 + 15 = 18

Sum of first ten terms S_{10} = (n/2)[2a + (n-1)d]

= (10/2)[2(3) + (10-1)(5)]

= 5[6 + 9(5)]

= 5[6 + 45]

= 5 × 51

= 255

Answer: Second term = 8, fourth term = 18, sum of first ten terms = 255

(b) A certain geometric progression has a common ratio of 2 and the sum of the first five terms is 155. Find the first term and give the formula for the n^{th} term

Common ratio r = 2, sum of first five terms S_5 = 155

$S_5 = a(1 - r^5) / (1 - r)$

$155 = a(1 - 2^5) / (1 - 2)$

155 = a(1 - 32) / (-1)

155 = a(-31) / (-1)

155 = 31a

a = 155 / 31 = 5

First term a = 5

Formula for the nth term: $T_n = ar^{n-1}$

$T_n = 5(2)^{n-1}$

Answer: First term = 5, nth term = $5(2)^{n-1}$

9. FIGURE 2 represents plotting of two stations A and B which are 4000m apart. T is a stationary target in the same vertical plane as A and B. When the distance from station A is 10,000m, the angle of elevation 30°.

(a) Calculate

(i) The vertical height of the target, TX

In triangle TAX (right-angled at X):

AX = 4000 m, AT = 10,000 m

AX is the adjacent side to the 30° angle, AT is the hypotenuse.

cos 30° = AX / AT

$\sqrt{3}/2 = 4000 / 10,000$

This does not hold, indicating we need to use the correct triangle. Use angle of elevation:

In triangle ATX, angle at A = 30°, AT = 10,000 m, TX is opposite.

sin 30° = TX / AT

1/2 = TX / 10,000

TX = 10,000 / 2 = 5,000 m

Answer: TX = 5,000 m

(ii) The distance AX, BX and TB

AX: From above, re-evaluate using triangle ATX:

cos 30° = AX / AT

$\sqrt{3}/2 = AX / 10,000$

$AX = 10,000 \times \sqrt{3}/2 = 5,000\sqrt{3} \approx 8,660$ m

BX:

$BX = AB + AX = 4{,}000 + 5{,}000\sqrt{3} \approx 4{,}000 + 8{,}660 = 12{,}660$ m

TB: In triangle TBX (right-angled at X):

$TX = 5{,}000$ m, $BX = 12{,}660$ m

$TB = \sqrt{TX^2 + BX^2}$

$= \sqrt{5{,}000^2 + 12{,}660^2}$

$= \sqrt{25{,}000{,}000 + 160{,}275{,}600}$

$= \sqrt{185{,}275{,}600}$

$\approx 13{,}612$ m

Answer: AX ≈ 8,660 m, BX ≈ 12,660 m, TB ≈ 13,612 m

(b) The angle of elevation of the target, T, from B

In triangle TBX:

$\tan \theta = TX / BX$

$= 5{,}000 / 12{,}660$

≈ 0.3949

$\theta = \tan^{-1}(0.3949) \approx 21.54°$

Answer: Angle of elevation ≈ 21.54°

10. (a) Find the solution of the quadratic equation $8x^2 - 34x + 21 = 0$ by using the factorization method

$8x^2 - 34x + 21 = 0$

We need two numbers that multiply to $8 \times 21 = 168$ and add to -34.

The numbers are -28 and -6:

$-28 + (-6) = -34$

$-28 \times -6 = 168$

Rewrite the equation:

$8x^2 - 28x - 6x + 21 = 0$

$4x(2x - 7) - 3(2x - 7) = 0$

$(2x - 7)(4x - 3) = 0$

$2x - 7 = 0 \rightarrow x = 7/2 = 3.5$

$4x - 3 = 0 \rightarrow x = 3/4 = 0.75$

Answer: $x = 3.5$ or $x = 0.75$

(b) Solve for x if $1/x + 2/(x-1) = 4$

Combine the fractions:

The common denominator is $x(x-1)$.

$1/x + 2/(x-1) = [1(x-1) + 2x] / [x(x-1)]$

$= (x - 1 + 2x) / [x(x-1)]$

$= (3x - 1) / [x(x-1)]$

Set equal to 4:

$(3x - 1) / [x(x-1)] = 4$

$3x - 1 = 4x(x-1)$

$3x - 1 = 4x^2 - 4x$

$0 = 4x^2 - 4x - 3x + 1$

$4x^2 - 7x + 1 = 0$

Solve using the quadratic formula $x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$, where $a = 4$, $b = -7$, $c = 1$:

$x = [7 \pm \sqrt{((-7)^2 - 4(4)(1))}] / (2(4))$

$= [7 \pm \sqrt{(49 - 16)}] / 8$

$= [7 \pm \sqrt{33}] / 8$

$x = (7 + \sqrt{33}) / 8$ or $x = (7 - \sqrt{33}) / 8$

Answer: $x = (7 + \sqrt{33}) / 8$ or $x = (7 - \sqrt{33}) / 8$

11. The number of units of proteins and starch contained in each of two types of food A and B are shown in the table below:

Type of Food | Units of Protein Per kg | Units of Starch Per kg | Cost per kg

A | 8 | 10 | 400/=

B | 12 | 6 | 500/=

Minimum Daily Requirement | 32 | 22 |

What is the cheapest way of satisfying the minimum daily requirement?

Let x be the kg of food A and y be the kg of food B.

Constraints:

Protein: $8x + 12y \geq 32 \rightarrow 4x + 6y \geq 16$ (divide by 2)

Starch: $10x + 6y \geq 22 \rightarrow 5x + 3y \geq 11$ (divide by 2)

$x \geq 0, y \geq 0$

Cost to minimize: $C = 400x + 500y$

Find the feasible region:

$4x + 6y \geq 16 \rightarrow 2x + 3y \geq 8$

$x = 0$: $3y = 8 \rightarrow y = 8/3 \approx 2.67$

$y = 0$: $2x = 8 \rightarrow x = 4$

$5x + 3y \geq 11$

$x = 0$: $3y = 11 \rightarrow y = 11/3 \approx 3.67$

$y = 0$: $5x = 11 \rightarrow x = 11/5 = 2.2$

Intersection of $2x + 3y = 8$ and $5x + 3y = 11$:

Subtract: $(5x + 3y) - (2x + 3y) = 11 - 8$

$3x = 3$

$x = 1$

Substitute $x = 1$ into $2x + 3y = 8$:

$2(1) + 3y = 8$

$3y = 6$

$y = 2$

Vertex: (1, 2)

Vertices of the feasible region: (4, 0), (1, 2), (0, 11/3).

Evaluate C:

(4, 0): C = 400(4) + 500(0) = 1600

(1, 2): C = 400(1) + 500(2) = 400 + 1000 = 1400

(0, 11/3): C = 400(0) + 500(11/3) = 500 × 11/3 = 5500/3 ≈ 1833.33

The cheapest way is at (1, 2).

Answer: Buy 1 kg of food A and 2 kg of food B for a total cost of shs. 1400.

12. The following table gives the scores of sixty students in a Basic Mathematics test.

Scores | Frequency

0 - 10 | 5

10 - 20 | 7

20 - 30 | 15

30 - 40 | 25

40 - 50 | 8

Calculate:

(a) The mean score if the assumed mean is obtained from the mid mark of the modal class

Modal class (highest frequency) is 30 - 40, frequency = 25.

Mid mark of modal class = (30 + 40) / 2 = 35

Assumed mean A = 35.

Class intervals:

0 - 10: Midpoint = 5, d = (5 - 35) / 10 = -3, f = 5, fd = 5(-3) = -15

10 - 20: Midpoint = 15, d = (15 - 35) / 10 = -2, f = 7, fd = 7(-2) = -14

20 - 30: Midpoint = 25, d = (25 - 35) / 10 = -1, f = 15, fd = 15(-1) = -15

30 - 40: Midpoint = 35, d = (35 - 35) / 10 = 0, f = 25, fd = 0

40 - 50: Midpoint = 45, d = (45 - 35) / 10 = 1, f = 8, fd = 8(1) = 8

Total f = 60, Total fd = -15 + (-14) + (-15) + 0 + 8 = -36

Mean = A + (Σfd / Σf) × class width

= 35 + (-36 / 60) × 10

= 35 - 0.6 × 10

= 35 - 6 = 29

Answer: Mean = 29

(b) The median

Cumulative frequency:

0 - 10: 5

10 - 20: 5 + 7 = 12

20 - 30: 12 + 15 = 27

30 - 40: 27 + 25 = 52

40 - 50: 52 + 8 = 60

Median position = (60 + 1) / 2 = 30.5th value, lies in the 30 - 40 class.

Median = L + [(n/2 - cf) / f] × c

L = 30 (lower boundary), n/2 = 30, cf = 27 (cumulative frequency before), f = 25 (frequency of median class), c = 10 (class width)

Median = 30 + [(30 - 27) / 25] × 10

= 30 + (3 / 25) × 10

= 30 + 1.2 = 31.2

Answer: Median = 31.2

(c) The range

Range = Highest score - Lowest score

= 50 - 0 = 50

Answer: Range = 50

13. In figure 3, ABCD is a rectangle in which AB = 3cm, BC = 2cm. V is a point such that AV = BV = CV = DV = 6cm, and AO = OC. Find:

(a) The angle VAD

Place the rectangle on a coordinate plane:

A(0,0), B(3,0), C(3,2), D(0,2).

AO = OC, so O is the midpoint of AC:

A(0,0), C(3,2) → O = (1.5, 1).

V(x, y) satisfies:

$AV = \sqrt{(x^2 + y^2)} = 6$

$BV = \sqrt{((x-3)^2 + y^2)} = 6$

$CV = \sqrt{((x-3)^2 + (y-2)^2)} = 6$

$DV = \sqrt{(x^2 + (y-2)^2)} = 6$

From AV: $x^2 + y^2 = 36$

From DV: $x^2 + (y-2)^2 = 36$

$x^2 + y^2 - 4y + 4 = 36$

$36 - 4y + 4 = 36$

$-4y + 4 = 0$

$y = 1$

$x^2 + 1 = 36$

$x^2 = 35$

$x = \pm\sqrt{35}$

Consider $x = -\sqrt{35}$ (since V is likely outside to the left):

$V = (-\sqrt{35}, 1)$

Vectors: $VA = A - V = (0 - (-\sqrt{35}), 0 - 1) = (\sqrt{35}, -1)$

$VD = D - V = (0 - (-\sqrt{35}), 2 - 1) = (\sqrt{35}, 1)$

Dot product $VA \cdot VD = (\sqrt{35})(\sqrt{35}) + (-1)(1) = 35 - 1 = 34$

Magnitudes: $|VA| = \sqrt{(35 + 1)} = 6$, $|VD| = \sqrt{(35 + 1)} = 6$

$\cos \theta = (VA \cdot VD) / (|VA||VD|) = 34 / (6 \times 6) = 34/36 = 17/18$

$\theta = \cos^{-1}(17/18) \approx 19.19°$

Answer: Angle VAD ≈ 19.19°

(b) The length of AC

$AC = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{9 + 4} = \sqrt{13}$

Answer: Length of AC = $\sqrt{13}$ cm

(c) The angle between VAD and the plane ABCD

Plane ABCD is the xy-plane (z = 0).

VAD lies in the plane formed by points V, A, D. Normal to VAD plane:

Vectors VA = $(\sqrt{35}, -1, 0)$, VD = $(\sqrt{35}, 1, 0)$

Cross product VA × VD = $(0, 0, (\sqrt{35})(1) - (\sqrt{35})(-1)) = (0, 0, 2\sqrt{35})$

Normal to ABCD = (0, 0, 1)

Angle between normals:

$\cos \varphi = (0, 0, 2\sqrt{35}) \cdot (0, 0, 1) / (|2\sqrt{35}| \times |1|) = 2\sqrt{35} / (2\sqrt{35}) = 1$

$\varphi = 0°$, so VAD is parallel to ABCD.

Answer: Angle = 0°

14. Study the given trial balance and answer questions that follow:

Trial Balance as of 31 December 2007

S/N	Details	Amount (Tshs)	Amount (Tshs)
1.	Cash	185,000.00	
2.	Capital		200,000.00
3.	Purchases	110,000.00	
4.	Sales		104,000.00
5.	Water bills	3,000.00	
6.	Advertising	2,000.00	
7.	Telephone bills	1,000.00	
8.	Salaries	3,000.00	
Total		304,000.00	304,000.00

Prepare the following for the year ending 31 December 2007:

(a) Trading account

Trading Account for the year ending 31 December 2007

Details | Amount (Tshs) | Details | Amount (Tshs)

Purchases | 110,000 | Sales | 104,000

Gross Loss | | 6,000

Total | 110,000 | Total | 110,000

Answer: Gross Loss = 6,000 Tshs

(b) Profit and loss account

Profit and Loss Account for the year ending 31 December 2007

Details | Amount (Tshs) | Details | Amount (Tshs)

Gross Loss | 6,000 | |

Water bills | 3,000 | |

Advertising | 2,000 | |

Telephone bills | 1,000 | |

Salaries | 3,000 | |

Net Loss | | 15,000

Total | 15,000 | Total | 15,000

Answer: Net Loss = 15,000 Tshs

(c) Balance sheet

Balance Sheet as of 31 December 2007

Assets | Amount (Tshs) | Liabilities | Amount (Tshs)

Cash | 185,000 | Capital | 200,000

| | Less: Net Loss | 15,000

| | Adjusted Capital | 185,000

Total | 185,000 | Total | 185,000