THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2012**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) By using mathematical tables, evaluate $\sqrt[3]{(0.007225/0.8140)^{1/2}}$ to three significant figures.

Answer:

$(0.007225/0.8140)^{1/2} = \sqrt{(0.007225/0.8140)}$

$= \sqrt{(0.0088759)} \approx 0.09419$ (using approximate square root)

$\sqrt[3]{(0.09419)} \approx 0.455$ (cube root approximation)

To 3 significant figures: 0.455

Final Answer: 0.455

(b) Rationalize $2/\sqrt{(1-\sqrt{3})}$

Answer:

$2/\sqrt{(1-\sqrt{3})} \times \sqrt{(1+\sqrt{3})}/\sqrt{(1+\sqrt{3})} = 2\sqrt{(1+\sqrt{3})}/(1-\sqrt{3})(1+\sqrt{3})$

$= 2\sqrt{(1+\sqrt{3})}/(1-3) = 2\sqrt{(1+\sqrt{3})}/-2 = -\sqrt{(1+\sqrt{3})}$

Final Answer: $-\sqrt{(1+\sqrt{3})}$

2. (a) Find the value of x for which $2^x * 16 = ½$

Answer:

$2^x * 16 = ½$

$16 = 2^4$

$2^x * 2^4 = 2^{-1}$

$2^{x+4} = 2^{-1}$

$x + 4 = -1$

$x = -5$

Final Answer: $x = -5$

(b) Solve $\log_5(x^2 + 3) - \log_5 x = 2\log_5 2$

Answer:

$\log_5(x^2 + 3) - \log_5 x = 2\log_5 2$

$\log_5((x^2 + 3)/x) = \log_5(2^2)$

$(x^2 + 3)/x = 4$

$x^2 + 3 = 4x$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

Final Answer: $x = 1$ or $x = 3$

3. (a) Mr. Bean lived a quarter of his life as a child, a fifth as a teenager and a third as an adult. He then spent 13 years in his old age. How old was he when he died?

Answer:

Let Mr. Bean's age = x

Child: $x/4$

Teenager: $x/5$

Adult: $x/3$

Old age: 13

$x/4 + x/5 + x/3 + 13 = x$

$(15x + 12x + 20x)/60 + 13 = x$

$47x/60 + 13 = x$

$13 = x - 47x/60$

$13 = 13x/60$

$x = 13 \times 60/13 = 60$

Final Answer: 60 years

(b) A and B are subsets of the universal set U. Find $n(A\cap B)$ given that $n(A) = 39$, $n(A\cap B') = 4$, $n(B') = 24$ and $n(U) = 65$.

Answer:

$n(A\cap B') = 4 \rightarrow n(A) - n(A\cap B) = 4$

$39 - n(A\cap B) = 4$

$n(A\cap B) = 35$

Final Answer: 35

4. Given that a = (3, 4), b = (1, -4) and c = (5, 2) determine:

(a) d = a - 4b + 2c

(b) magnitude of vector d, leaving your answer in the form $m\sqrt{n}$;

(c) the direction cosines of d and hence show that the sum of the squares of these direction cosines is one.

Answer:

(a) d = a - 4b + 2c

a = (3, 4), b = (1, -4), c = (5, 2)

d = (3, 4) - 4(1, -4) + 2(5, 2)

= (3, 4) - (4, -16) + (10, 4)

= (3 - 4 + 10, 4 + 16 + 4) = (9, 24)

(b) Magnitude of d = $\sqrt{(9^2 + 24^2)} = \sqrt{(81 + 576)} = \sqrt{657} = \sqrt{(9\times73)} = 3\sqrt{73}$

(c) Direction cosines: $\cos \theta_x = 9/\sqrt{657}$, $\cos \theta_\gamma = 24/\sqrt{657}$

Sum of squares: $(9/\sqrt{657})^2 + (24/\sqrt{657})^2 = (81 + 576)/657 = 657/657 = 1$

Final Answer: (a) (9, 24), (b) $3\sqrt{73}$, (c) $\cos \theta_x = 9/\sqrt{657}$, $\cos \theta_\gamma = 24/\sqrt{657}$, sum = 1

5. (a) If polygons X and Y are similar and their areas are $16cm^2$ and the corresponding side of polygon X is $28cm^2$, what is the length of a side of polygon Y if the corresponding side of polygon X is 8cm?

Answer:

Area ratio = 16/28 = 4/7

Side ratio = $\sqrt{(4/7)} = 2/\sqrt{7}$

Side of X = 8 cm

Side of Y = $8 \times (2/\sqrt{7}) = 16/\sqrt{7}$ cm

Final Answer: $16/\sqrt{7}$ cm

(b) (i) Show whether triangles PQR and ABC are similar or not

Answer:

ΔPQR: PQ = 10 cm, QR = 15 cm, PR = 25 cm, ∠Q = 40°, ∠R = 100°

ΔABC: AB = 6 cm, BC = 9 cm, ∠B = 40°, ∠C = 100°

PQ/AB = 10/6 = 5/3

QR/BC = 15/9 = 5/3

∠Q = ∠B = 40°, ∠R = ∠C = 100°

Similar by SAS (side ratios equal, included angle equal).

Final Answer: Triangles are similar

(b) (ii) Find the relationship between y cm and x cm in the triangles given above.

Answer:

ΔPQR ~ ΔABC

PQ/AB = QR/BC = PR/AC

PR = 25 cm, AB = 6 cm, BC = 9 cm

PR/AC = PQ/AB = 5/3

25/AC = 5/3

AC = 25 × 3/5 = 15 cm

Let y = AC = 15 cm, x = PR = 25 cm

y/x = 15/25 = 3/5

y = (3/5)x

Final Answer: y = (3/5)x

6. (a) The power (P) used in an electric circuit is directly proportional to the square of the current (I). When the current is 8 Ampere (A), the power used is 640 Watts (W).

(i) write down the equation relating the power (P) and the current (I),

(ii) calculate the current (I) when the circuit uses 360 Watts.

Answer:

(i) $P \propto I^2$

$P = kI^2$

$640 = k(8)^2$

$640 = 64k$

$k = 10$

$P = 10I^2$

(ii) $360 = 10I^2$

$I^2 = 36$

$I = \pm 6$

Current = 6 A (discard negative)

Final Answer: (i) $P = 10I^2$, (ii) 6 A

(b) If x * y is defined as ½(x + y), find (5 * -2) * (3 * -4).

Answer:

x * y = ½(x + y)

(5 * -2) = ½(5 + -2) = ½(3) = 3/2

(3 * -4) = ½(3 + -4) = ½(-1) = -1/2

(3/2 * -1/2) = ½(3/2 + -1/2) = ½(1) = 1/2

Final Answer: 1/2

7. (a) By selling an article at shs. 22,500/- a shopkeeper makes a loss of 10%. At what price must the shopkeeper sell the article in order to get a profit of 10%?

Answer:

Selling price = 90% of cost price

$22500 = 0.9 \times CP$

CP = 22500/0.9 = 25000

To get 10% profit:

SP = 1.1 × 25000 = 27500

Final Answer: sh 27500

(b) An alloy consists of three metals A, B and C in the proportion A : B = 3 : 5 and B : C = 7 : 6. Calculate the proportion A : C.

Answer:

A : B = 3 : 5

B : C = 7 : 6

A : B : C = (3×7) : (5×7) : (5×6) = 21 : 35 : 30

A : C = 21 : 30 = 7 : 10

Final Answer: 7 : 10

8. (a) If the 5^{th} term of an arithmetic progression is 23 and the 12^{th} term is 37, find the first term and the common difference.

Answer:

5^{th} term: a + 4d = 23 (1)

12^{th} term: a + 11d = 37 (2)

Subtract (1) from (2):

(a + 11d) - (a + 4d) = 37 - 23

7d = 14

d = 2

Substitute d = 2 in (1):

a + 4×2 = 23

a + 8 = 23

a = 15

Final Answer: First term = 15, common difference = 2

(b) Find the sum of the first four terms of a geometric progression which has a first term of 1 and a common ratio of ½.

Answer:

$a = 1$, $r = 1/2$

Terms: 1, 1/2, 1/4, 1/8

Sum = 1 + 1/2 + 1/4 + 1/8 = (8 + 4 + 2 + 1)/8 = 15/8

Final Answer: 15/8

9. (a) Find the length AC from the figure below:

Answer:

ΔABC: $\angle ABC = 86°$, $\angle BAC = 26°$, BC = 22.2 cm

$\angle ACB = 180° - 86° - 26° = 68°$

Law of Sines: AC/sin 86° = 22.2/sin 26°

sin 86° ≈ 0.9976, sin 26° ≈ 0.4384

AC/0.9976 = 22.2/0.4384

AC = (22.2 × 0.9976)/0.4384 ≈ 50.5 cm

Final Answer: 50.5 cm

(b) A ladder reaches the top of a wall 18m high when the other end on the ground is 8m from the wall. Find the length of the ladder.

Answer:

Right triangle:

Height = 18 m, base = 8 m

Length = $\sqrt{(18^2 + 8^2)} = \sqrt{(324 + 64)} = \sqrt{388} = 2\sqrt{97}$ m

Final Answer: $2\sqrt{97}$ m

10. (a) Solve for x if ½ = 1 - ⅓

Answer:

Equation seems incomplete: ½ = 1 - ⅓

$\frac{1}{2} = \frac{2}{3}$ (not true), assume typo, possibly $\frac{1}{2}x = 1 - \frac{1}{3}x$

$\frac{1}{2}x + \frac{1}{3}x = 1$

$(3x + 2x)/6 = 1$

$5x/6 = 1$

$x = 6/5$

Final Answer: $x = 6/5$ (assuming corrected equation)

(b) If the sum of two numbers is 3 and the sum of their squares is 29, find the numbers.

Answer:

$x + y = 3$ (1)

$x^2 + y^2 = 29$ (2)

From (1): $y = 3 - x$

Substitute in (2):

$x^2 + (3 - x)^2 = 29$

$x^2 + 9 - 6x + x^2 = 29$

$2x^2 - 6x - 20 = 0$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x = 5$ or $x = -2$

$x = 5, y = -2$

$x = -2, y = 5$

Final Answer: 5 and -2, or -2 and 5

11. Anna and Mary are tailors. They make x blouses and y skirts each week. Anna does all the cutting and Mary does all the sewing. To make a blouse it takes 5 hours of cutting and 4 hours of sewing. To make a skirt it takes 6 hours of cutting and 10 hours of sewing. Neither tailor works more than 60 hours a week.

(a) For sewing show that $2x + 5y \leq 30$

Since Mary does all the sewing, we calculate her total sewing hours for x blouses and y skirts. Each blouse requires 4 hours of sewing, and each skirt requires 10 hours of sewing. Mary's total sewing time is:

$4x + 10y$

Mary cannot work more than 60 hours per week. The problem states the inequality for sewing should be in the form $2x + 5y \leq 30$. Simplify the expression:

$4x + 10y \leq 60$

Divide through by 2:

$(4x + 10y)/2 \leq 60/2$

$2x + 5y \leq 30$

Thus, the inequality $2x + 5y \leq 30$ is shown.

(b) Write down another inequality in x and y for the cutting

Anna does all the cutting. Each blouse requires 5 hours of cutting, and each skirt requires 6 hours of cutting. Anna’s total cutting time for x blouses and y skirts is:

$5x + 6y$

Anna cannot work more than 60 hours per week, so the inequality for cutting is:

$5x + 6y \leq 60$

(c) IF they make at least 8 blouses each week, write down another inequality

If they make at least 8 blouses each week, the number of blouses x must satisfy:

$x \geq 8$

(d) Using 1 cm to represent 1 unit on each axis, show the information in parts (a), (b) and (c) graphically. Shade only the required region.

To graph the inequalities:

$2x + 5y \leq 30$ (sewing):

When $x = 0$: $5y = 30 \rightarrow y = 6 \rightarrow$ point (0, 6)

When $y = 0$: $2x = 30 \rightarrow x = 15 \rightarrow$ point (15, 0)

Line: $2x + 5y = 30$, shade below the line.

$5x + 6y \leq 60$ (cutting):

When $x = 0$: $6y = 60 \rightarrow y = 10 \rightarrow$ point (0, 10)

When $y = 0$: $5x = 60 \rightarrow x = 12 \rightarrow$ point (12, 0)

Line: $5x + 6y = 60$, shade below the line.

$x \geq 8$:

Vertical line at $x = 8$, shade to the right.

(e) IF the profit on a blouse is shs. 3,000/= and on a skirt is shs. 10,000/=, calculate the maximum profit that Anna and Mary can make in a week

The profit function to maximize is:

$P = 3000x + 10000y$

We need to evaluate P at the vertices of the feasible region determined by the inequalities:

$2x + 5y \leq 30$

$5x + 6y \leq 60$

$x \geq 8$

$x \geq 0, y \geq 0$

Find the vertices by solving the intersections:

Intersection of $2x + 5y = 30$ and $5x + 6y = 60$:

Multiply $2x + 5y = 30$ by 5: $10x + 25y = 150$

Multiply $5x + 6y = 60$ by 2: $10x + 12y = 120$

Subtract: $(10x + 25y) - (10x + 12y) = 150 - 120$

$13y = 30$

$y = 30/13 \approx 2.31$

Substitute $y = 30/13$ into $2x + 5y = 30$:

$2x + 5(30/13) = 30$

$2x + 150/13 = 30$

$2x = 30 - 150/13 = 390/13 - 150/13 = 240/13$

$x = 120/13 \approx 9.23$

Vertex: (120/13, 30/13)

Intersection of 2x + 5y = 30 and x = 8:

2(8) + 5y = 30

16 + 5y = 30

5y = 14

y = 14/5 = 2.8

Vertex: (8, 2.8)

Intersection of 5x + 6y = 60 and x = 8:

5(8) + 6y = 60

40 + 6y = 60

6y = 20

y = 20/6 = 10/3 ≈ 3.33

Vertex: (8, 10/3)

Evaluate P at the vertices:

At (120/13, 30/13):

P = 3000(120/13) + 10000(30/13) = (360000 + 300000)/13 = 660000/13 ≈ 50769.23

At (8, 2.8):

P = 3000(8) + 10000(2.8) = 24000 + 28000 = 52000

At (8, 10/3):

P = 3000(8) + 10000(10/3) = 24000 + 100000/3 ≈ 24000 + 33333.33 = 57333.33

The maximum profit is approximately shs. 57,333.33, achieved when they make 8 blouses and 10/3 skirts.

12. In a survey of the number of children in 12 houses, the following data resulted: 1, 2, 3, 4, 2, 1, 3, 4, 3, 5, 3, 2

(a) Show this data in a frequency distribution table

Data: 1, 2, 3, 4, 2, 1, 3, 4, 3, 5, 3, 2

Frequency distribution table:

Number of children	Frequency
1	2
2	3
3	4
4	2
5	1

(b) Draw a histogram and a frequency polygon to represent this data

To draw a histogram:

X-axis: Number of children (1 to 5).

Y-axis: Frequency.

Bars: Width 1 unit, heights are 2, 3, 4, 2, 1 for 1, 2, 3, 4, 5 children, respectively.

To draw a frequency polygon:

Plot points at the midpoints of the histogram bars at their respective frequencies: (1, 2), (2, 3), (3, 4), (4, 2), (5, 1).

Connect the points with straight lines.

For closure, add points (0, 0) and (6, 0) at the ends.

(c) Calculate the mean and mode number of children per house

Mean:

Sum of data = 1 + 2 + 3 + 4 + 2 + 1 + 3 + 4 + 3 + 5 + 3 + 2 = 33

Number of houses = 12

Mean = 33/12 = 2.75

Mode:

From the frequency table, the number 3 has the highest frequency (4).

Mode = 3

13. (a) An open rectangular box measures externally 32cm long, 27cm wide and 15cm deep. IF the box is made of wood 1cm thick, find the volume of wood used

External dimensions: 32 cm long, 27 cm wide, 15 cm deep.

Thickness of wood: 1 cm.

Internal dimensions (subtract 2 cm from each dimension due to 1 cm thickness on both sides):

Length = 32 - 2 = 30 cm

Width = 27 - 2 = 25 cm

Depth = 15 - 1 = 14 cm (only subtract 1 cm since the box is open on top)

External volume = 32 × 27 × 15 = 12960 cm^3

Internal volume = 30 × 25 × 14 = 10500 cm^3

Volume of wood = External volume - Internal volume = 12960 - 10500 = 2460 cm^3

The volume of wood used is 2460 cm^3.

(b) Find the distance (in km) between towns P(12°4'S, 30°5'E) and Q(12°4'S, 39°8'E) along a line of latitude, correctly to 4 decimal places

Both towns are on the same latitude (12°4'S), so we calculate the distance along the latitude using the difference in longitude.

Longitude of P = 30°5' = 30 + 5/60 = 30.0833°

Longitude of Q = 39°8' = 39 + 8/60 = 39.1333°

Difference in longitude = 39.1333 - 30.0833 = 9.05°

The distance along a latitude is given by:

Distance = (difference in longitude in degrees) × cos(latitude) × 111 km

Latitude = 12°4' = 12 + 4/60 = 12.0667°

cos(12.0667°) ≈ 0.9780

Distance = 9.05 × 0.9780 × 111 ≈ 982.2951 km

The distance is 982.2951 km, correct to 4 decimal places.

14. (a) The following balances were extracted from the ledgers of Mr. and Mrs. Mkono business on 31st January. Prepare a trial balance.

Capital 30,000/=

Furniture 25,000/=

Motor vehicle 45,000/=

Sales 65,000/=

Purchases 54,000/=

Creditors 76,000/=

Debtors 15,000/=

Insurance 3,000/=

Cash 18,000/=

Discount received 7,000/=

Discount allowed 4,000/=

Drawing 12,000/=

Electricity 5,000/=

Trial Balance as of 31st January:

Account	Debit (shs.)	Credit (shs.)
Capital		30,000
Furniture	25,000	
Motor vehicle	45,000	
Sales		65,000
Purchases	54,000	
Creditors		76,000
Debtors	15,000	
Insurance	3,000	
Cash	18,000	
Discount received		7,000
Discount allowed	4,000	

Drawing | 12,000 |

Electricity | 5,000 |

Total | 181,000 | 178,000

The trial balance does not balance; there is a discrepancy of 181,000 - 178,000 = 3,000, indicating a possible error in the records.

(b) Determine the gross profit and the net profit from the information given below

Sales 38,000/=

Opening stock 8,000/=

Purchases 25,000/=

Electricity 4,000/=

Discount allowed 2,000/=

Closing stock 5,000/=

Gross Profit:

Cost of goods sold (COGS) = Opening stock + Purchases - Closing stock

COGS = 8000 + 25000 - 5000 = 28000

Gross Profit = Sales - COGS = 38000 - 28000 = 10000

Net Profit:

Net Profit = Gross Profit - Expenses

Expenses = Electricity + Discount allowed = 4000 + 2000 = 6000

Net Profit = 10000 - 6000 = 4000

Gross Profit = shs. 10,000

Net Profit = shs. 4,000

15. (a) Find the value of k such that the matrix [2k + 2 k; 4k - 3 k + 3] is singular

A matrix is singular if its determinant is 0. For the matrix [2k+2 k; 4k-3 k+3]:

Determinant = (2k+2)(k+3) - (k)(4k-3)

= (2k+2)(k+3) - k(4k-3)

$= 2k(k+3) + 2(k+3) - 4k^2 + 3k$

$= 2k^2 + 6k + 2k + 6 - 4k^2 + 3k$

$= (2k^2 - 4k^2) + (6k + 2k + 3k) + 6$

$= -2k^2 + 11k + 6$

Set the determinant to 0:

$-2k^2 + 11k + 6 = 0$

Multiply by -1:

$2k^2 - 11k - 6 = 0$

Solve the quadratic equation using the quadratic formula $k = [-b \pm \sqrt{(b^2 - 4ac)}]/(2a)$, where a = 2, b = -11, c = -6:

$k = [11 \pm \sqrt{((-11)^2 - 4(2)(-6))}]/(2(2))$

$= [11 \pm \sqrt{(121 + 48)}]/4$

$= [11 \pm \sqrt{169}]/4$

$= [11 \pm 13]/4$

$k = (11 + 13)/4 = 24/4 = 6$

$k = (11 - 13)/4 = -2/4 = -0.5$

The values of k are 6 and -0.5.

(b) The vertices of Δ ABC are A(1, 2), B(3, 1) and C(-2, 1). IF triangle ABC is reflected on the x-axis, find the coordinates of its image

Reflection over the x-axis changes the y-coordinate to its negative while the x-coordinate remains the same: (x, y) → (x, -y).

A(1, 2) → A'(1, -2)

B(3, 1) → B'(3, -1)

C(-2, 1) → C'(-2, -1)

The coordinates of the image are A'(1, -2), B'(3, -1), C'(-2, -1).

(c) Solve the following simultaneous equations by matrix method

$2x + 3y = 2$

$-6y + 8x - 1 = 0$

Rewrite the second equation:

$-6y + 8x - 1 = 0 \rightarrow 8x - 6y = 1$

In matrix form:

[2 3; 8 -6][x; y] = [2; 1]

Let A = [2 3; 8 -6], X = [x; y], B = [2; 1].

Solve for X using $X = A^{-1}B$.

Determinant of A = (2)(-6) - (3)(8) = -12 - 24 = -36

Adjoint of A = [-6 -3; -8 2] (transpose of cofactor matrix)

A^{-1} = (1/det(A)) × adjoint(A) = (1/-36)[-6 -3; -8 2] = [6/36 3/36; 8/36 -2/36] = [1/6 1/12; 2/9 -1/18]

Multiply A^{-1} by B:

X = [1/6 1/12; 2/9 -1/18][2; 1]

x = (1/6)(2) + (1/12)(1) = 2/6 + 1/12 = 4/12 + 1/12 = 5/12

y = (2/9)(2) + (-1/18)(1) = 4/9 - 1/18 = 8/18 - 1/18 = 7/18

Solution: x = 5/12, y = 7/18.

16. A box contains 7 red balls and 14 black balls. Two balls are drawn at random without replacement.

(a) Draw a tree diagram to show the results of the drawing

First draw:

Probability of red = 7/21 = 1/3

Probability of black = 14/21 = 2/3

Second draw (if first is red):

Total balls left = 20

Red left = 6, Black left = 14

Probability of red = 6/20 = 3/10

Probability of black = 14/20 = 7/10

Second draw (if first is black):

Total balls left = 20

Red left = 7, Black left = 13

Probability of red = 7/20

Probability of black = 13/20

Tree diagram:

First level: Red (1/3), Black (2/3)

Second level (from Red): Red (3/10), Black (7/10)

Second level (from Black): Red (7/20), Black (13/20)

I cannot draw the diagram directly, but this outlines the structure.

(b) Find the probability that both are black

Probability (both black) = Probability (first black) × Probability (second black | first black)

= (14/21) × (13/20)

= (2/3) × (13/20)

= 26/60 = 13/30

The probability that both are black is 13/30.

(c) Find the probability that they are of the same colour

Probability (same colour) = Probability (both red) + Probability (both black)

Both red = (7/21) × (6/20) = (1/3) × (3/10) = 3/30 = 1/10

Both black = 13/30 (from part b)

Total = 1/10 + 13/30 = 3/30 + 13/30 = 16/30 = 8/15

The probability that they are of the same colour is 8/15.

(d) Find the probability that the first is black and the second is red

Probability (first black, second red) = Probability (first black) × Probability (second red | first black)

= (14/21) × (7/20)

= (2/3) × (7/20)

= 14/60 = 7/30

The probability that the first is black and the second is red is 7/30.

(e) Verify the probability rule P(A) + P(A') = 1 by using the results in part (b)

Let A be the event "both balls are black."

P(A) = 13/30 (from part b)

A' is the event "both balls are not black" (i.e., at least one is red).

P(A') = 1 - P(A) = 1 - 13/30 = 17/30

Verify: P(A) + P(A') = 13/30 + 17/30 = 30/30 = 1

The rule P(A) + P(A') = 1 is verified.

Prepared by: Maria Marco for TETEA