THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2014**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) Kisiki and Jembe are riding on a circular path. Kisiki completes a round in 24 minutes whereas Jembe completes a round in 36 minutes. If they both started at the same place and time and go in the same direction, after how many minutes will they meet again at the starting point?

Answer:

Time for Kisiki = 24 minutes

Time for Jembe = 36 minutes

They meet at the starting point when both complete whole rounds (LCM of 24 and 36):

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

LCM = $2^3 \times 3^2 = 8 \times 9 = 72$

Final Answer: 72 minutes

(b) An empty bottle weighs 115 grams. If 45 tablets each weighing 3 grams are put in the bottle, what is the total weight?

Answer:

Weight of bottle = 115 grams

Weight of 1 tablet = 3 grams

Number of tablets = 45

Weight of tablets = $45 \times 3 = 135$ grams

Total weight = $115 + 135 = 250$ grams

Final Answer: 250 grams

2. (a) (i) Express $(3/5 + 5/3)^2$ in the form $a + b\sqrt{3}$, where a and b are integers.

Answer:

$(3/5 + 5/3) = (3/5 + 5/3)$

Common denominator: 15

$3/5 = 9/15$, $5/3 = 25/15$

$(9/15 + 25/15) = 34/15$

$(34/15)^2 = 34^2/15^2 = 1156/225$

Since 1156/225 cannot be expressed in the form $a + b\sqrt{3}$ (no $\sqrt{3}$ term arises), assume a possible typo. Let's try a related expression, e.g., $(3/5 + 5\sqrt{3})^2$:

$(3/5 + 5\sqrt{3})^2 = (3/5)^2 + 2(3/5)(5\sqrt{3}) + (5\sqrt{3})^2$

$= 9/25 + 2(3/5)(5\sqrt{3}) + 25\times3$

$= 9/25 + 30/5\sqrt{3} + 75$

$= 9/25 + 6\sqrt{3} + 75$

$= (9/25 + 75) + 6\sqrt{3}$

$= (9/25 + 1875/25) + 6\sqrt{3}$

$= 1884/25 + 6\sqrt{3}$

$a = 1884/25$, $b = 6$ (but a is not an integer, problem may expect different form)

Original expression doesn't yield $\sqrt{3}$, so final answer based on given:

Final Answer: 1156/225 (problem may have typo)

3. (a) (ii) Express $(3/5 + 5/3)^2$ in the form $p + q\sqrt{3}$, where p and q are rational numbers.

Answer:

$(3/5 + 5/3) = 34/15$ (as calculated above)

$(34/15)^2 = 1156/225$

$p = 1156/225$, $q = 0$ (since no $\sqrt{3}$ term)

Final Answer: $p = 1156/225$, $q = 0$

(b) Solve for x: $(81)^{1/4} \times 81 = \sqrt{9}^x$.

Answer:

$(81)^{1/4} \times 81 = \sqrt{9}^x$

$81 = 3^4$

$(81)^{1/4} = (3^4)^{1/4} = 3^1 = 3$

Left side: $3 \times 81 = 3 \times 3^4 = 3^5$

Right side: $\sqrt{9^x} = (9^x)^{1/2}$

$9 = 3^2$

$9^x = (3^2)^x = 3^{2x}$

$(9^x)^{1/2} = (3^{2x})^{1/2} = 3^x$

$3^5 = 3^x$

$x = 5$

Final Answer: $x = 5$

4. (a) The Venn diagram below shows the number of elements in sets P and Q.

[P: 38 + 2x, Intersection: x, Q: 13 + x]

If $n(P \cup Q) = 95$, calculate:

(i) The value of x.

(ii) $n(P \cap Q)$.

Answer:

(i) $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

$n(P) = 38 + 2x$

$n(Q) = 13 + x$

$n(P \cap Q) = x$

$95 = (38 + 2x) + (13 + x) - x$

$95 = 38 + 13 + 2x$

$95 = 51 + 2x$

$2x = 95 - 51 = 44$

$x = 44/2 = 22$

(ii) $n(P \cap Q) = x = 22$

Final Answer: (i) $x = 22$, (ii) 22

(b) The age of Timothy is ⅓ the age of his father. If the sum of their ages is 54 years, find the age of the father.

Answer:

Let father's age = F

Timothy's age = (1/3)F

F + (1/3)F = 54

(4/3)F = 54

F = 54 × 3/4 = 162/4 = 40.5

Final Answer: 40.5 years

5. (a) Find the equation of the line passing through (6, 4) and perpendicular to the line whose equation is 12x + 6y = 9.

Answer:

Given line: 12x + 6y = 9

6y = -12x + 9

y = -2x + 3/2

Slope = -2

Perpendicular slope = 1/2

Line through (6, 4):

y - 4 = (1/2)(x - 6)

y - 4 = (1/2)x - 3

y = (1/2)x + 1

Final Answer: y = (1/2)x + 1

(b) If a = 2i + 3j, b = 19i - 15j and c = 5i - 7j, find the value of x such that xa + yb = c.

Answer:

xa + yb = c

$x(2i + 3j) + y(19i - 15j) = 5i - 7j$

$(2x + 19y)i + (3x - 15y)j = 5i - 7j$

Equate coefficients:

$2x + 19y = 5$ (1)

$3x - 15y = -7$ (2)

Multiply (1) by 15, (2) by 19:

$30x + 285y = 75$ (3)

$57x - 285y = -133$ (4)

Add (3) and (4):

$87x = 75 - 133 = -58$

$x = -58/87 = -2/3$

Substitute $x = -2/3$ into (1):

$2(-2/3) + 19y = 5$

$-4/3 + 19y = 5$

$19y = 5 + 4/3 = 19/3$

$y = (19/3) / 19 = 1/3$

Final Answer: $x = -2/3$

6. (a) Given that XY = 2 cm, BC = 3 cm and area of XYCB = 10 cm², calculate the area of triangle AXY.

Answer:

XYCB is a quadrilateral with XY parallel to BC (assumed parallelogram for area formula).

Area of parallelogram XYCB = XY × distance between XY and BC = 10 cm²

Distance = 10 / XY = 10 / 2 = 5 cm

Triangle AXY:

Base XY = 2 cm

Height = distance from A to XY, assume same as parallelogram height = 5 cm

Area = (1/2) × base × height = (1/2) × 2 × 5 = 5 cm^2

Final Answer: 5 cm^2

(b) Determine the length of one side of a regular quadrilateral inscribed in a circle of radius 10 cm.

Answer:

Regular quadrilateral = square inscribed in circle

Radius = 10 cm

Diagonal of square = diameter = 2 × 10 = 20 cm

Side s of square: diagonal = $s\sqrt{2}$

$20 = s\sqrt{2}$

$s = 20/\sqrt{2} = 20\sqrt{2}/2 = 10\sqrt{2}$

Final Answer: $10\sqrt{2}$ cm

7. (a) Juma sells one litre of milk at sh 600. How many litres of milk will Juma sell to get sh 208,800?

Answer:

Cost per litre = 600 sh

Total amount = 208800 sh

Litres = 208800 / 600 = 348

Final Answer: 348 litres

(b) The compression f of a spring is directly proportional to the thrust, T newtons exerted on it. If a thrust of 2 newtons produces a compression of 0.4 cm, find:

(i) The compression when the thrust is 5 newtons.

(ii) The thrust when the compression is 0.7 cm.

Answer:

$f \propto T$

$f = kT$

$0.4 = k \times 2$

k = 0.4/2 = 0.2

f = 0.2T

(i) T = 5

f = 0.2 × 5 = 1 cm

(ii) f = 0.7

0.7 = 0.2T

T = 0.7/0.2 = 3.5 newtons

Final Answer: (i) 1 cm, (ii) 3.5 newtons

8. (a) Kieku has to share 80 books with his younger sisters Upendo and Okuli. He decided that for every 2 books that Okuli gets, Upendo gets 3 and he gets 5 books. Find the number of books each gets.

Answer:

Ratio Okuli:Upendo:Kieku = 2:3:5

Total parts = 2 + 3 + 5 = 10

Total books = 80

Okuli = (2/10) × 80 = 16

Upendo = (3/10) × 80 = 24

Kieku = (5/10) × 80 = 40

Final Answer: Okuli: 16, Upendo: 24, Kieku: 40

(b) Nyauwa invested a certain amount of money in a bank which pays interest rate of 6 percent after every 6 months. After 5 years, his total savings were sh 9,600,000. Determine the amount of money Nyauwa invested initially.

Answer:

Compound interest, compounded semi-annually:

A = P(1 + r/n)^(nt)

A = 9600000

r = 0.06, n = 2 (semi-annually), t = 5 years

nt = 2 × 5 = 10

r/n = 0.06/2 = 0.03

$9600000 = P(1 + 0.03)^{10}$

$(1.03)^{10} \approx 1.3439$

9600000 = P × 1.3439

P = 9600000 / 1.3439 ≈ 7143400

Final Answer: sh 7143400

(a) The 20th term of an arithmetic progression is 60 and the 16th term is 20. Find the sum of the first 40 terms.

Answer:

nth term: a + (n-1)d

20th term: a + 19d = 60 (1)

16th term: a + 15d = 20 (2)

Subtract (2) from (1):

(a + 19d) - (a + 15d) = 60 - 20

4d = 40

d = 10

Substitute d = 10 into (2):

a + 15×10 = 20

a + 150 = 20

a = -130

Sum of first 40 terms:

$S_n = n/2\ [2a + (n-1)d]$

$S_{40} = 40/2\ [2(-130) + (40-1)\times 10]$

= 20 [-260 + 39×10]

= 20 [-260 + 390]

$= 20 \times 130 = 2600$

Final Answer: 2600

(b) A shopkeeper invested sh 4,800,000 for 5 years. If the amount of money accumulated is sh 7,730,450, calculate the compound interest rate.

Answer:

$A = P(1 + r)^t$

$A = 7730450$, $P = 4800000$, $t = 5$

$7730450 = 4800000\,(1 + r)^5$

$(1 + r)^5 = 7730450 / 4800000 \approx 1.6105$

$1 + r = (1.6105)^{1/5} \approx 1.100$

$r \approx 0.100 = 10\%$

Final Answer: 10%

9. (a) Find the length marked y in four significant figures.

Answer:

In triangle ABC:

$\angle ABC = 45°$, $\angle ACB = 30°$, $BC = 6$ cm

Use the Law of Sines: $a/\sin A = b/\sin B = c/\sin C$

$\angle BAC = 180° - 45° - 30° = 105°$

$AB = c$, $BC = a = 6$ cm, $AC = b = y$

$a/\sin A = b/\sin B$

$6/\sin 105° = y/\sin 30°$

$\sin 105° = \sin (60° + 45°) = \sin 60° \cos 45° + \cos 60° \sin 45°$

$= (\sqrt{3}/2)(\sqrt{2}/2) + (1/2)(\sqrt{2}/2) = (\sqrt{6} + \sqrt{2})/4 \approx 0.9659$

$\sin 30° = 0.5$

$6/0.9659 = y/0.5$

$y = (6 \times 0.5)/0.9659 \approx 3.106$

To 4 significant figures: 3.106

Final Answer: 3.106 cm

(b) A 4-m ladder rests against a vertical wall with its foot 2 m from the wall. How far up the wall does the ladder reach? Give your answer in two decimal places.

Answer:

Ladder forms a right triangle:

Hypotenuse (ladder) = 4 m

Base (distance from wall) = 2 m

Height (up the wall) = h

Pythagorean theorem:

$h^2 + 2^2 = 4^2$

$h^2 + 4 = 16$

$h^2 = 12$

$h = \sqrt{12} = \sqrt{(4\times3)} = 2\sqrt{3} \approx 2 \times 1.732 = 3.464$

To 2 decimal places: 3.46

Final Answer: 3.46 m

10. (a) Use the quadratic formula to solve $x^2 + 4x - 12 = 0$

Answer:

$x^2 + 4x - 12 = 0$

$a = 1, b = 4, c = -12$

Quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}]/(2a)$

Discriminant: $b^2 - 4ac = 4^2 - 4(1)(-12) = 16 + 48 = 64$

$x = [-4 \pm \sqrt{64}]/(2\times1) = [-4 \pm 8]/2$

$x = (4 + 8)/2 = 2$ or $x = (-4 - 8)/2 = -6$

Final Answer: $x = 2$ or $x = -6$

(b) A garden measuring 12 m by 16 m is to have a pedestrian pathway of equal width constructed all around it, increasing the total area to 285 square metres. What will be the width of the pathway?

Answer:

Garden: $12 \text{ m} \times 16 \text{ m}$

Area $= 12 \times 16 = 192 \text{ m}^2$

Pathway width $= w$

New dimensions: $(12 + 2w) \times (16 + 2w)$

New area $= (12 + 2w)(16 + 2w) = 285$

$(12 + 2w)(16 + 2w) = 285$

$192 + 24w + 32w + 4w^2 = 285$

$4w^2 + 56w + 192 = 285$

$4w^2 + 56w - 93 = 0$

$w^2 + 14w - 23.25 = 0$

Quadratic formula: $w = [-14 \pm \sqrt{(14^2 - 4(1)(-23.25))}]/(2\times1)$

$= [-14 \pm \sqrt{(196 + 93)}]/2$

$= [-14 \pm \sqrt{289}]/2$

$= [-14 \pm 17]/2$

$w = 3/2 = 1.5$ or $w = -31/2$ (discard negative)

Final Answer: 1.5 m

11. A farmer has 20 hectares for growing tomatoes and cabbages. The cost per hectare for tomatoes is sh 480,000 and for cabbages is sh 120,000. The farmer has budgeted sh 768,000. Tomatoes require 48 man-days per hectare and cabbages require two man-days per hectare. There are 36 man-days available. The profit on tomatoes is sh 160,000 per hectare and on cabbages is sh 192,000 per hectare. Find the number of hectares of each crop the farmer should plant to maximize the profit.

Answer:

Let x = hectares of tomatoes, y = hectares of cabbages

Constraints:

$x + y \leq 20$ (land)

$480000x + 120000y \leq 768000 \rightarrow 4x + y \leq 6.4$ (cost)

$48x + 2y \leq 36 \rightarrow 24x + y \leq 18$ (man-days)

$x \geq 0, y \geq 0$

Profit: $P = 160000x + 192000y$

Vertices of feasible region:

(0, 0): P = 0

(0, 6.4): $P = 192000 \times 6.4 = 1228800$

(0.75, 0): $24x + y = 18 \rightarrow 24 \times 0.75 + 0 = 18$, $4x + y = 4 \times 0.75 + 0 = 3 \leq 6.4$

$P = 160000 \times 0.75 = 120000$

(0.5, 3.4): $24x + y = 18 \rightarrow 24 \times 0.5 + y = 18 \rightarrow y = 6$, but $4x + y = 4 \times 0.5 + 6 = 8 > 6.4$, adjust:

$4x + y = 6.4$, $24x + y = 18 \rightarrow 20x = 11.6 \rightarrow x = 0.58$, $y = 4.08$

$P = 160000 \times 0.58 + 192000 \times 4.08 = 92800 + 783360 = 876160$

Max profit at (0, 6.4)

Final Answer: 0 hectares tomatoes, 6.4 hectares cabbages

12. The heights of some plants grown in a laboratory were recorded after 5 weeks. The results are shown in the following table:

Height (cm)	11 – 15	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40
Frequency	4	8	20	21	13	3

(a) Calculate the mean and mode

(b) Draw a cumulative frequency curve for the data

(c) Estimate the median from the graph

Answer:

(a) Mean:

Midpoints: 13, 18, 23, 28, 33, 38

Sum = (13×4) + (18×8) + (23×20) + (28×21) + (33×13) + (38×3)

= 52 + 144 + 460 + 588 + 429 + 114 = 1787

Total frequency = 4 + 8 + 20 + 21 + 13 + 3 = 69

Mean = 1787/69 ≈ 25.8986 ≈ 25.9

Mode: Modal class 26–30 (frequency 21)

(b) Cumulative frequencies:

≤15: 4

≤20: 4 + 8 = 12

≤25: 12 + 20 = 32

≤30: 32 + 21 = 53

≤35: 53 + 13 = 66

≤40: 66 + 3 = 69

points are (15, 4), (20, 12), (25, 32), (30, 53), (35, 66), (40, 69)

(c) Median: 69/2 = 34.5th value, in 26–30 class

Median ≈ 25 + (34.5 - 32)/21 × 5 ≈ 25 + 0.595 = 25.6

Final Answer: (a) Mean: 25.9 cm, Mode: 26–30, (b) Cumulative frequencies listed, (c) 25.6 cm

13. (a) Prove that the sizes of the angles in the same segment of a circle are equal.

Answer:

Consider circle with center O, chord AB, points C and D on major arc AB.

∠ACB and ∠ADB are angles in the same segment (major arc AB).

Inscribed angle theorem: Angle subtended by an arc at the circumference is half the angle subtended at the center.

∠AOB is the central angle for arc AB.

∠ACB = (1/2)∠AOB

$\angle ADB = (1/2)\angle AOB$

Thus, $\angle ACB = \angle ADB$.

Final Answer: Angles in the same segment are equal (proved)

(b) In the figure below, O is the centre of the circle and AD bisects angle BAC. Find angle BCD.

Answer:

AD bisects $\angle BAC$, so $\angle BAD = \angle CAD$.

In ΔABC, AD intersects BC at D.

Assume typical circle properties (e.g., AB = AC, making ΔABC isosceles).

Since O is the center, OA = OB = OC (radii).

If AB = AC, then $\angle ABC = \angle ACB$ (base angles of isosceles triangle).

$\angle BAC + \angle ABC + \angle ACB = 180°$.

Without specific angles or figure details, assume cyclic quadrilateral ABCD:

$\angle BCD + \angle BAD = 180°$ (opposite angles of cyclic quadrilateral).

Need more data to compute exact angle (e.g., $\angle BAC$ or other angles).

Final Answer: Cannot determine without additional angle data

(c) Kicheko and Mkauja are two villages on latitude 60°S. The distance between Kicheko and Mkauja measured along the parallel of latitude is 1111 km. Find the difference between their longitudes in two significant figures.

Answer:

Latitude = 60°S

Distance along parallel = 1111 km

Earth's radius R = 6400 km (from previous questions)

Distance $s = R \times \theta \times \cos(\text{latitude})$

θ in radians

$1111 = 6400 \times \theta \times \cos 60°$

$\cos 60° = 0.5$

$1111 = 6400 \times \theta \times 0.5$

$\theta = 1111/(6400\times0.5) = 1111/3200 \approx 0.3472$ radians

Convert to degrees: $\theta = 0.3472 \times (180/\pi) \approx 19.89° \approx 20°$ (2 significant figures)

Final Answer: 20°

14. Mr. Kijembe started business on 16th March 2011 with capital in cash 2,066,000/=

March 17 bought goods for Cash 1,116,000/=

19 purchased goods for Cash 300,100/=

20 sold goods for Cash 800,000/=

21 sold goods for Cash 1,400,000/=

26 paid Rent 300,000/=

Record the above transactions in a cash account ledger and extract a trial balance. State two uses of the trial balance you have prepared.

Answer:

Cash Account:

Date	Details	Amount	Date	Details	Amount
Mar 16	Capital	2066000	Mar 17	Purchases	1116000
Mar 20	Sales	800000	Mar 19	Purchases	300100
Mar 21	Sales	1400000	Mar 26	Rent	300000
		Mar 31	Balance c/d	2349900	
Total		4266000	Total		4266000

Trial Balance as of 31 March 2011:

Account | Debit | Credit

Capital | | 2066000

Purchases | 1416100 |

Sales | | 2200000

Rent | 300000 |

Cash | 2349900 |

Total | 4066000 | 4266000

Uses of trial balance:

To check the arithmetic accuracy of ledger entries (debits = credits).

To prepare financial statements like balance sheet and income statement.

Final Answer: Trial balance prepared, uses: check accuracy, prepare financial statements

15. (a) (i) Determine a matrix M which represents a reflection in the line $y^2 - x = 0$ after a reflection in the line $y - x = 0$.

Answer:

$y - x = 0 \rightarrow y = x$: Reflection matrix $M_1 = [0\ 1;\ 1\ 0]$

$y^2 - x = 0 \rightarrow y^2 = x$: Not a line, assume typo, possibly $x^2 - y = 0 \rightarrow x^2 = y$

Assume second reflection in $y = -x$: $M_2 = [0\ -1;\ -1\ 0]$

Combined reflection: $M = M_2 \times M_1$

$M_1 = [0\ 1;\ 1\ 0]$

$M_2 = [0\ -1;\ -1\ 0]$

$M = [0\ -1;\ -1\ 0] \times [0\ 1;\ 1\ 0] = [0\times0 + (-1)\times1\ \ 0\times1 + (-1)\times0;\ (-1)\times0 + 0\times1\ \ (-1)\times1 + 0\times0]$

$= [-1\ 0;\ 0\ -1]$

Final Answer: $M = [-1\ 0;\ 0\ -1]$

(a) (ii) If $A = [3\ 2]$

$[-1\ 4]$, find $|A|$ and A^{-1}.

Answer:

$A = [3\ 2;\ -1\ 4]$

$|A| = (3\times4) - (2\times(-1)) = 12 + 2 = 14$

$A^{-1} = (1/|A|) \times adj(A)$

adj(A) = [4 -2; 1 3]

A^{-1} = (1/14) × [4 -2; 1 3] = [4/14 -2/14; 1/14 3/14] = [2/7 -1/7; 1/14 3/7]

Final Answer: |A| = 14, A^{-1} = [2/7 -1/7; 1/14 3/7]

(b) (i) Use the inverse matrix obtained in b(i) to solve 3x + 2y = 12

4x - y = 5

Answer:

Equations: 3x + 2y = 12, -x + 4y = 5

Matrix form: [3 2; -1 4][x; y] = [12; 5]

[x; y] = A^{-1} [12; 5]

A^{-1} = [2/7 -1/7; 1/14 3/7]

[x; y] = [2/7 -1/7; 1/14 3/7] [12; 5]

x = (2/7)×12 + (-1/7)×5 = 24/7 - 5/7 = 19/7

y = (1/14)×12 + (3/7)×5 = 12/14 + 15/7 = 6/7 + 15/7 = 21/7 = 3

Final Answer: x = 19/7, y = 3

16. (a) A bag contains 6 white balls and 3 yellow balls. A ball is selected at random and not replaced. Another ball is then selected. Find the probability of selecting one white ball and one yellow ball.

Answer:

Total balls = 6 white + 3 yellow = 9

White then Yellow: (6/9) × (3/8) = 18/72 = 1/4

Yellow then White: (3/9) × (6/8) = 18/72 = 1/4

Total probability = 1/4 + 1/4 = 1/2

Final Answer: 1/2

(b) Given f(x) = { 4 when $x \leq 1$

{ 5 when $1 < x \leq 2$

{ x when $x > 2$

(i) Sketch the graph of $f(x)$.

(ii) State the range of $f(x)$.

(iii) Is $f(x)$ a one-to-one function? Give reason(s).

Answer:

(i) Description

$x \leq 1$: $y = 4$ (horizontal line)

$1 < x \leq 2$: $y = 5$ (horizontal line)

$x > 2$: $y = x$ (line starting at (2, 2))

(ii) Range: $y = 4$ for $x \leq 1$, $y = 5$ for $1 < x \leq 2$, $y > 2$ for $x > 2$

Range = $\{4\} \cup \{5\} \cup (2, \infty) = \{4, 5\} \cup (2, \infty)$

(iii) Not one-to-one: $f(x) = 4$ for all $x \leq 1$ (multiple x values map to 4), $f(x) = 5$ for $1 < x \leq 2$ (multiple x values map to 5).

Final Answer: (i) Described graph, (ii) $\{4, 5\} \cup (2, \infty)$, (iii) Not one-to-one, multiple x map to same y you