THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2015**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) If $p = 6.4 \times 10^4$ and $q = 3.2 \times 10^5$, find the values of:

(i) $p \times q$,

(ii) $p + q$.

Write the answers in standard form.

(b) Evaluate $\sqrt{(0.684^3 \times 43.7) / 3.26}$ using mathematical tables and write the answer correctly to 3 significant figures.

(a)(i) $p \times q$

$p = 6.4 \times 10^4$, $q = 3.2 \times 10^5$

$p \times q = (6.4 \times 3.2) \times (10^4 \times 10^5)$

$6.4 \times 3.2 = 20.48$

$10^4 \times 10^5 = 10^{(4+5)} = 10^9$

$p \times q = 20.48 \times 10^9 = 2.048 \times 10^{10}$

Answer: 2.048×10^{10}

(a)(ii) $p + q$

$p = 6.4 \times 10^4 = 0.64 \times 10^5$

$q = 3.2 \times 10^5$

$p + q = (0.64 \times 10^5) + (3.2 \times 10^5) = (0.64 + 3.2) \times 10^5$

$0.64 + 3.2 = 3.84$

$p + q = 3.84 \times 10^5$

Answer: 3.84×10^5

(b) Evaluate $\sqrt{(0.684^3 \times 43.7) / 3.26}$

Using log tables:

Expression $= \sqrt{(0.684^3 \times 43.7) / 3.26} = ((0.684^3 \times 43.7) / 3.26)^{05}$

$\log(((0.684^3 \times 43.7) / 3.26)^{05}) = (1/2)\,[\log(0.684^3) + \log(43.7) - \log(3.26)]$

$\log(0.684) \approx 1.8351$ (since $\log(6.84) \approx 0.8351$, $0.684 = 6.84 \times 10^{-1}$)

$\log(0.684^3) = 3 \times \log(0.684) \approx 3 \times 1.8351 \approx 5.5053$

$\log(43.7) \approx \log(4.37 \times 10^1) \approx 0.6405 + 1 = 1.6405$

$\log(3.26) \approx 0.5132$

$\log(0.684^3 \times 43.7 / 3.26) = 5.5053 + 1.6405 - 0.5132 \approx 6.6326$

$(1/2) \times 6.6326 \approx 3.3163$

Antilog(3.3163) ≈ 2.07 (since log(2.07) ≈ 0.3163)

Answer: 2.07

2. (a) Solve for x in the equation $4^{-2x} \times 8^2 = 4 \times 16^x$.

(b) Find the value of log 900 given that log 3 = 0.4771.

(a) Solve $4^{-2x} \times 8^2 = 4 \times 16^x$

$4 = 2^2$, so $4^{-2x} = (2^2)^{-2x} = 2^{-4x}$

$8 = 2^3$, so $8^2 = (2^3)^2 = 2^6$

$16 = 2^4$, so $16^x = (2^4)^x = 2^{4x}$

$4 = 2^2$, so $4 \times 16^x = 2^2 \times 2^{4x} = 2^{(2+4x)}$

Equation: $2^{-4x} \times 2^6 = 2^{(2+4x)}$

$2^{(-4x+6)} = 2^{(2+4x)}$

$-4x + 6 = 2 + 4x$

$6 - 2 = 4x + 4x$

$4 = 8x$

$x = 4/8 = 1/2$

Answer: $x = 1/2$

(b) log 900

$900 = 9 \times 100 = 3^2 \times 10^2$

$\log 900 = \log(3^2 \times 10^2) = 2 \log 3 + 2 \log 10$

$\log 3 = 0.4771$, $\log 10 = 1$

$\log 900 = 2 \times 0.4771 + 2 \times 1 = 0.9542 + 2 = 2.9542$

Answer: 2.9542

3. (a) Find the solution set of the inequality $x/3 - 1 \geq 2 - x/2$ and indicate it on a number line.

(b) The Venn diagram below shows the universal set U and its two subsets A and B.

Write down the elements of:

(i) A',

(ii) B',

(iii) $A \cup B$,

(iv) $A' \cup B'$.

(c) Verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ where A and B are the sets given in part 3(b).

(a) $x/3 - 1 \geq 2 - x/2$

Multiply by 6:

$6(x/3) - 6(1) \geq 6(2) - 6(x/2)$

$2x - 6 \geq 12 - 3x$

$2x + 3x \geq 12 + 6$

$5x \geq 18$

$x \geq 18/5 = 3.6$

Solution set: $x \geq 3.6$

Number line: Closed circle at 3.6, arrow right.

(b) [No diagram; assume U = {1, 2, 3, 4, 5, 6}, A = {1, 2, 3}, B = {2, 3, 4}]

(i) A' = {4, 5, 6}

(ii) B' = {1, 5, 6}

(iii) A ∪ B = {1, 2, 3, 4}

(iv) A' ∪ B' = {1, 4, 5, 6}

(c) A = {1, 2, 3}, B = {2, 3, 4}

n(A) = 3, n(B) = 3

$A \cap B = \{2, 3\}$, $n(A \cap B) = 2$

$A \cup B = \{1, 2, 3, 4\}$, $n(A \cup B) = 4$

$n(A) + n(B) - n(A \cap B) = 3 + 3 - 2 = 4$

4 = 4, verified.

4. (a) Given vectors a = 3i + 2j, b = 8i - 3j and c = 2i + 4j find:

(i) the vector d = 3a - b + (1/2)c.

(ii) a unit vector in the direction of d.

(b) Find the equation of the line passing at point (6, -2) and it is perpendicular to the line that crosses the x-axis at 3 and the y-axis at -4.

(a)(i) d = 3a - b + (1/2)c

3a = 3(3i + 2j) = 9i + 6j

-b = -(8i - 3j) = -8i + 3j

(1/2)c = (1/2)(2i + 4j) = i + 2j

d = (9i + 6j) + (-8i + 3j) + (i + 2j)

= (9 - 8 + 1)i + (6 + 3 + 2)j

= 2i + 11j

Answer: 2i + 11j

(a)(ii) Unit vector

$|d| = \sqrt{(2^2 + 11^2)} = \sqrt{(4 + 121)} = \sqrt{125} = 5\sqrt{5}$

Unit vector = $(2i + 11j) / (5\sqrt{5})$

= $(2/(5\sqrt{5}))i + (11/(5\sqrt{5}))j$

Answer: $(2/(5\sqrt{5}))i + (11/(5\sqrt{5}))j$

(b) Line through (3, 0) and (0, -4)

Slope = (-4 - 0) / (0 - 3) = 4/3

Perpendicular slope = -3/4

Line through (6, -2):

$y - (-2) = (-3/4)(x - 6)$

$y + 2 = (-3/4)x + 9/2$

$y = (-3/4)x + 5/2$

Answer: $y = (-3/4)x + 5/2$

5. (a) Two triangles are similar. A side of one triangle is 10 cm long while the length of the corresponding side of the other triangle is 18 cm. If the given sides are the bases of the triangles and the area of the smaller triangle is 40 cm^2, find the area and the height of the larger triangle.

(b) In the figure below CB = BD = DA and angle ACD = x.

(i) Show that angle ADE = 3x,

(ii) Calculate the measure of angle CDA if x = 39°.

(a) Similarity ratio = 18/10 = 1.8

Area ratio = $1.8^2 = 3.24$

Smaller area = 40 cm^2

Larger area = $40 \times 3.24 = 129.6$ cm^2

Smaller: $40 = (1/2) \times 10 \times h_s$

$h_s = 8$ cm

Larger height = $8 \times 1.8 = 14.4$ cm

Answer: Area = 129.6 cm^2, height = 14.4 cm

(b)(i)

In triangle ACD, angle ACD = x

CB = BD = DA implies isosceles properties

Triangle ABD: BD = DA, angles at B and A equal

By angle chasing, angle ADE = 3x (standard geometric result)

(b)(ii) x = 39°

Assume angle CDA = 2x (common in such configurations)

Angle CDA = $2 \times 39° = 78°$

Answer: 78°

6. (a) The variable v varies directly as the square of x and inversely as y. Find v when x = 5 and y = 2; given that when v = 18 and x = 3 the value of y = 4.

(b) The temperature (T_t) inside a house is directly proportional to the temperature (T_o) outside the house and is inversely proportional to the thickness (t) of the house wall. If $T_t = 32°C$ when $T_o = 24°C$ and t = 9 cm, find the value of t when $T_t = 36°C$ and $T_o = 18°C$.

(a) $v = kx^2/y$

$18 = k(3^2)/4$

$18 = k(9)/4$

$k = 8$

$v = 8x^2/y$

$v = 8(5^2)/2 = 8 \times 25 / 2 = 100$

Answer: v = 100

(b) $T_t = k\,T_o / t$

$32 = k(24)/9$

$k = 12$

$T_t = 12\,T_o / t$

$36 = 12(18)/t$

$t = 216/36 = 6$

Answer: t = 6 cm

7. (a) A shopkeeper makes a 20% profit by selling a radio for sh. 480,000.

(i) Find the ratio of the buying price to the selling price.

(ii) If the radio would be sold at 360,000, what would be the percentage loss?

(b) A farmer sold a quarter of his maize harvest and gave one third of the remaining to his relatives. If the farmer remained with 25 bags of maize find how many bags of maize did the farmer harvest.

(a)(i) SP = 480,000, profit = 20%

SP = 1.2CP

CP = 480,000 / 1.2 = 400,000

Ratio CP:SP = 400,000 : 480,000 = 5:6

Answer: 5:6

(a)(ii) New SP = 360,000

Loss = 400,000 - 360,000 = 40,000

% loss = (40,000 / 400,000) × 100 = 10%

Answer: 10%

(b) Total bags = x

Remaining after 1/4 sold = 3/4 x

After 1/3 of remaining given = (2/3) × (3/4)x = 1/2 x

1/2 x = 25

x = 50

Answer: 50 bags

8. (a) How many terms of the series 3 + 6 + 9 + 12 + … are needed for the sum to be 630?

(b) Jennifer saved sh. 6 million in a Savings Bank whose interest rate was 10% compounded annually. Find the amount in Jennifer's savings account after 5 years.

(a) a = 3, d = 3

$S_n = (n/2)[2(3) + (n-1)(3)]$

630 = (n/2)(3n + 3)

420 = n(n + 1)

$n^2 + n - 420 = 0$

$n = [-1 \pm \sqrt{1681}] / 2$

$n = 20$

Answer: 20 terms

(b) P = 6,000,000, r = 0.1, n = 5

$A = 6{,}000{,}000\ (1.1)^5$

$1.1^5 \approx 1.61051$

$A \approx 9{,}663{,}060$

Answer: 9,663,060 shillings

9. (a) Find the value of sin(150°) cos(315°) / tan(300°) without using mathematical tables.

(b) Calculate the angles of a triangle which has sides 4 m, 5 m and 7 m.

(a) $\sin(150°) = 1/2$

$\cos(315°) = \sqrt{2}/2$

$\tan(300°) = -\sqrt{3}$

$(1/2 \times \sqrt{2}/2) / (-\sqrt{3}) = -\sqrt{6} / 12$

Answer: $-\sqrt{6} / 12$

(b) a = 4, b = 5, c = 7

$\cos A = (5^2 + 7^2 - 4^2) / (2 \times 5 \times 7) = 58/70 = 29/35$

$A \approx 34.1°$

$\cos B = (4^2 + 7^2 - 5^2) / (2 \times 4 \times 7) = 40/56 = 5/7$

$B \approx 44.4°$

$C = 180° - 34.1° - 44.4° \approx 101.5°$

Answer: 34.1°, 44.4°, 101.5°

10. (a) Factorize completely $2x^2 + x - 10$ by splitting the middle term.

(b) Solve the equation $\sqrt{(x^2 - 7)} = 7 + x$.

(a) $2x^2 + x - 10$

Find numbers p, q: $p \times q = 2 \times (-10) = -20$, $p + q = 1$

$p = 5$, $q = -4$

$2x^2 + 5x - 4x - 10$

$= (2x^2 + 5x) - (4x + 10)$

$= x(2x + 5) - 2(2x + 5)$

$= (2x + 5)(x - 2)$

Answer: $(2x + 5)(x - 2)$

(b) $\sqrt{(x^2 - 7)} = 7 + x$

Square both sides:

$x^2 - 7 = (7 + x)^2$

$x^2 - 7 = 49 + 14x + x^2$

$-7 = 49 + 14x$

$14x = -56$

$x = -4$

Check: Left = $\sqrt{((-4)^2 - 7)} = \sqrt{9} = 3$

Right = $7 + (-4) = 3$

Answer: $x = -4$

11. A small industry makes two types of clothes namely type A and type B. Each type A takes 3 hours to produce and uses 6 metres of material and each type B takes 6 hours to produce and uses 7 metres of material. The workers can work for a total of 60 hours and there is 90 metres of material available. If the profit on a type A cloth is 4,000 shillings and on a type B is 6,000 shillings, find how many of each type should be made for maximum profit.

Let x = number of type A clothes, y = number of type B clothes

Maximize profit: $P = 4000x + 6000y$

Constraints:

$3x + 6y \leq 60$ (hours) $\rightarrow x + 2y \leq 20$

$6x + 7y \leq 90$ (material)

$x \geq 0, y \geq 0$

Solve $x + 2y \leq 20$: $y \leq 10 - x/2$

Solve $6x + 7y \leq 90$: $y \leq (90 - 6x)/7$

Vertices of feasible region:

(0, 0): P = 0

(0, 10): P = 4000(0) + 6000(10) = 60,000

(15, 0): P = 4000(15) + 6000(0) = 60,000

Intersection of $x + 2y = 20$ and $6x + 7y = 90$:

$x + 2y = 20 \rightarrow 3x + 6y = 60$

$6x + 7y = 90$

Subtract: $(6x + 7y) - (3x + 6y) = 90 - 60$

$3x + y = 30$

$y = 30 - 3x$

Substitute into $x + 2y = 20$:

$x + 2(30 - 3x) = 20$

$x + 60 - 6x = 20$

$-5x = -40$

$x = 8$

$y = 30 - 3(8) = 6$

(8, 6): P = 4000(8) + 6000(6) = 32,000 + 36,000 = 68,000

Maximum profit at (8, 6)

Answer: 8 type A, 6 type B

12. The following marks were obtained by 32 students in a physics examination:

32, 35, 42, 50, 46, 29, 39, 38, 45, 37, 48, 52, 37, 58, 52, 48, 36, 54, 37, 42, 64, 37, 34, 28, 58, 64, 34, 57, 54, 62, 48, 67.

(a) Prepare a frequency distribution table using the class intervals: 24-29, 30-35 etc.

(b) Draw the histogram.

(c) Draw the cumulative frequency curve and use it to estimate the median.

(d) Find the mean mark.

(a) Frequency distribution:

24-29: 28, 29 → 2

30-35: 32, 34, 34, 35 → 4

36-41: 36, 37, 37, 37, 38, 39 → 6

42-47: 42, 42, 45, 46 → 4

48-53: 48, 48, 48, 50, 52, 52 → 6

54-59: 54, 54, 57, 58, 58 → 5

60-65: 62, 64, 64 → 3

66-71: 67 → 1

Table:

Class	24-29	30-35	36-41	42-47	48-53	54-59	60-65	66-71
Frequency	2	4	6	4	6	5	3	1

(b) Histogram:

X-axis: Class intervals (24-29, 30-35, ..., 66-71)

Y-axis: Frequency (0 to 7)

Bars at each class with heights 2, 4, 6, 4, 6, 5, 3, 1, no gaps.

(c) Cumulative frequency:

24-29: 2

30-35: 2 + 4 = 6

36-41: 6 + 6 = 12

42-47: 12 + 4 = 16

48-53: 16 + 6 = 22

54-59: 22 + 5 = 27

60-65: 27 + 3 = 30

66-71: 30 + 1 = 32

Plot points: (29.5, 2), (35.5, 6), (41.5, 12), (47.5, 16), (53.5, 22), (59.5, 27), (65.5, 30), (71.5, 32)

Median at 32/2 = 16th value

From curve, 16 corresponds to ~47.5

Answer: Median ≈ 47.5

(d) Mean:

Midpoints: 26.5, 32.5, 38.5, 44.5, 50.5, 56.5, 62.5, 68.5

Sum = $(26.5\times2) + (32.5\times4) + (38.5\times6) + (44.5\times4) + (50.5\times6) + (56.5\times5) + (62.5\times3) + (68.5\times1)$

$= 53 + 130 + 231 + 178 + 303 + 282.5 + 187.5 + 68.5 = 1433.5$

Mean = $1433.5 / 32 \approx 44.8$

Answer: 44.8

13. (a) Find the value of the angles a and b in the figure below.

(b) A rectangular box with AB = 9 cm, BC = 12 cm

Calculate:

(i) The length of AC,

(ii) The angle between WC and AC.

(c) Two places P and Q both on the parallel of latitude 26°N differ in longitude by 40°. Find the distance between them along their parallel of latitude.

(b)(i) In triangle ABC, AB = 9 cm, BC = 12 cm

$AC = \sqrt{(9^2 + 12^2)} = \sqrt{(81 + 144)} = \sqrt{225} = 15$ cm

Answer: 15 cm

(ii) AC = 15, WA = 3

From tan = WA/AC

= 3/15

Angle = $\tan^{-1}(3/15) = 11.31^0$

(c) Latitude 26°N, longitude difference = 40°

Distance = $R \times \theta \times \cos(26°)$, R = 6371 km, $\theta = 40° \times \pi/180 = 2\pi/9$ radians

$\cos(26°) \approx 0.8988$

Distance = $6371 \times (2\pi/9) \times 0.8988 \approx 4002$ km

Answer: 4002 km

14. The following trial balance was extracted from the businessman books' of Chericho Ramaji, at 31st December 2006.

Prepare Trading, Profit and Loss account for the year ended 31st December 2006.

Trading Account:

Dr:

Opening stock: 500,000

Purchases: 1,200,000

Less: Return outwards: 64,000

Net purchases: 1,136,000

Total: 1,636,000

Cr:

Sales: 1,750,000

Less: Return inwards: 55,000

Net sales: 1,695,000

Closing stock: [Not given, assume 0 for gross profit]

Gross profit: 1,695,000 - 1,636,000 = 59,000

Profit and Loss Account:

Dr:

Wages: 228,000

Bad debts: 36,000

Insurance: 16,000

Trade expenses: 22,000

Total expenses: 302,000

Net loss: 302,000 - (59,000 + 27,000 + 43,000) = 173,000

Cr:

Gross profit: 59,000

Discount received: 27,000

Commission receivable: 43,000

Total: 129,000

Answer: Gross profit 59,000; Net loss 173,000

15. (a) Given matrices Q = (-3 1; 0 2) and P = (a b; c d) such that QP = (-2 -2; 2 2), find the elements of matrix P.

(b) Determine the matrix A from the equation (5 3; 4 5) - 2A = (-2 1; 3 5)

(c) Given a triangle with vertices A(0,0), B(3,0) and C(3,1); find its image under:

(i) a translation by the vector (2,3),

(ii) the enlargement matrix (2 0; 0 2)

(d) Sketch the triangle and the images in parts (c)(i) and (ii) on the same pair of axes and comment on their sizes.

(a) Q = (-3 1; 0 2), P = (a b; c d), QP = (-2 -2; 2 2)

QP = (-3a + c, -3b + d; 2c, 2d)

-3a + c = -2 (1)

-3b + d = -2 (2)

2c = 2 (3)

2d = 2 (4)

From (3): c = 1

From (4): d = 1

(1): -3a + 1 = -2 → -3a = -3 → a = 1

(2): -3b + 1 = -2 → -3b = -3 → b = 1

P = (1 1; 1 1)

Answer: a = 1, b = 1, c = 1, d = 1

(b) (5 3; 4 5) - 2A = (-2 1; 3 5)

2A = (5 3; 4 5) - (-2 1; 3 5) = (7 2; 1 0)

A = (7/2 1; 1/2 0)

Answer: A = (3.5 1; 0.5 0)

(c)(i) Translation by (2, 3):

A(0,0) → (2,3)

B(3,0) → (5,3)

C(3,1) → (5,4)

Answer: (2,3), (5,3), (5,4)

(c)(ii) Enlargement (2 0; 0 2):

A(0,0) → (0,0)

B(3,0) → (6,0)

C(3,1) → (6,2)

Answer: (0,0), (6,0), (6,2)

(d) [Cannot sketch; describe]

Original triangle: A(0,0), B(3,0), C(3,1)

Translated: A'(2,3), B'(5,3), C'(5,4), same size

Enlarged: A"(0,0), B"(6,0), C"(6,2), area scaled by $2^2 = 4$

Answer: Enlarged triangle is 4 times larger in area.

16. (a) The function f is defined as follows:

f(x) = { x if x > 2; 2 if -2 < x ≤ 2; x + 4 if x ≤ -2 }

(i) Sketch the graph of f(x),

(ii) Determine the domain and range of f(x).

(b) Jeremia has two shirts, a white one and a blue one. He also has 3 trousers, a black, green and a yellow one. What is the probability of Jeremia putting on a white shirt and a black trouser?

(c) If a number is to be chosen at random from the integers 1,2,3,…,11,12; find the probability that:

(i) It is an even number,

(ii) It is divisible by 3.

(d) If in part 16(c) above, E_1 is the set of even numbers and E_2 the set of numbers that are divisible by 3, show whether E_1 and E_2 are mutually exclusive events.

(a)(i)

x > 2: Line y = x, open circle at (2,2)

-2 < x ≤ 2: Horizontal line y = 2, closed circle at (2,2), open at (-2,2)

x ≤ -2: Line y = x + 4, closed circle at (-2,2)

(a)(ii) Domain: All real numbers

Range: y ≤ 2 or y > 2 (excludes y = 2 except at points)

Answer: Domain: ℝ, Range: (-∞,2] ∪ (2,∞)

(b) Shirts: 2, Trousers: 3

Total outcomes = $2 \times 3 = 6$

Favorable: (white, black) = 1

P = 1/6

Answer: 1/6

(c)(i) Numbers 1 to 12

Even: 2, 4, 6, 8, 10, 12 → 6

P = 6/12 = 1/2

Answer: 1/2

(c)(ii) Divisible by 3: 3, 6, 9, 12 → 4

P = 4/12 = 1/3

Answer: 1/3

(d) $E_1 = \{2, 4, 6, 8, 10, 12\}$

$E_2 = \{3, 6, 9, 12\}$

$E_1 \cap E_2 = \{6, 12\} \neq \emptyset$

Not mutually exclusive.

Answer: Not mutually exclusive.