THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2017**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Round off:

(i) 9.67 to ones,

(ii) 0.205 to one decimal place,

(iii) 0.0197 to two decimal places.

Hence, estimate the value of $(2.67 \times 0.205) / 0.0197$.

(b) Simplify the expressions:

(i) $(3 + \sqrt{2})(4 - 2\sqrt{2})$,

(ii) $\sqrt{40} \times \sqrt{45}$.

(c) Express 0.3636… in the form of a/b where a and b are integers and $b \neq 0$.

(a)(i) $9.67 \approx 10$

(ii) $0.205 \approx 0.2$

(iii) $0.0197 \approx 0.02$

Estimate: $(2.67 \times 0.205) / 0.0197 \approx (3 \times 0.2) / 0.02 = 0.6 / 0.02 = 30$

Answer: 30

(b)(i) $(3 + \sqrt{2})(4 - 2\sqrt{2}) = 12 - 6\sqrt{2} + 4\sqrt{2} - 4 = 12 - 2\sqrt{2} - 4 = 8 - 2\sqrt{2}$

Answer: $8 - 2\sqrt{2}$

(b)(ii) $\sqrt{40} \times \sqrt{45} = \sqrt{(40 \times 45)} = \sqrt{1800} = \sqrt{(36 \times 50)} = 6\sqrt{50} = 6 \times 5\sqrt{2} = 30\sqrt{2}$

Answer: $30\sqrt{2}$

(c) Let $x = 0.3636…$

$100x = 36.3636…$

$100x - x = 36.3636… - 0.3636…$

$99x = 36$

$x = 36/99 = 4/11$

Answer: 4/11

2. (a) Simplify:

(i) $27^{1/4} \times 3^{1/4} \times (\sqrt{3})^2$,

(ii) $\log_3 10 + \log_3 8.1$.

(b) If $n \log_5 125 = \log_2 64$, find the value of n.

(a)(i) $27 = 3^3$, so $27^{1/4} = (3^3)^{1/4} = 3^{3/4}$

$(\sqrt{3})^2 = 3$

$27^{1/4} \times 3^{1/4} \times 3 = 3^{3/4} \times 3^{1/4} \times 3^1 = 3^{(3/4+1/4+1)} = 3^2 = 9$

Answer: 9

(a)(ii) $\log_3 10 + \log_3 8.1 = \log_3 (10 \times 8.1) = \log_3 81$

$81 = 3^4$, so $\log_3 81 = 4$

Answer: 4

(b) $\log_5 125 = \log_5 (5^3) = 3$

$\log_2 64 = \log_2 (2^6) = 6$

$n \times 3 = 6$

$n = 2$

Answer: n = 2

3. (a) Factorize the following expressions:

(i) $16y^2 + xy - 15x^2$,

(ii) $4 - (3x - 1)^2$.

(b) At Moiva's graduation ceremony 45 people drank Pepsi-Cola, 80 drank Coca-Cola and 35 drank both Pepsi-Cola and Coca-Cola. By using a Venn diagram, found out how many people were at the ceremony if each person drank Pepsi-Cola or Coca-Cola.

(a)(i) $16y^2 + xy - 15x^2$

Find p, q: $p \times q = 16 \times (-15) = -240$, $p + q = 1$

$p = 16, q = -15$

$16y^2 + 16xy - 15xy - 15x^2 = (16y^2 + 16xy) - (15xy + 15x^2)$

$= 8y(2y + x) - 15x(y + x) = (2y + x)(8y - 15x)$

Answer: $(2y + x)(8y - 15x)$

(a)(ii) $4 - (3x - 1)^2$

$= (2)^2 - (3x - 1)^2$

$= [2 - (3x - 1)][2 + (3x - 1)]$

$= (2 - 3x + 1)(2 + 3x - 1) = (3 - 3x)(1 + 3x)$

Answer: $(3 - 3x)(1 + 3x)$

(b) Pepsi (P): 45, Coca-Cola (C): 80, $P \cap C$: 35

P only = 45 - 35 = 10

C only = 80 - 35 = 45

Total = P only + C only + $P \cap C$ = 10 + 45 + 35 = 90

Answer: 90 people

4. (a) Given the three vectors a = 4i + 6j, b = 4i + 10j and c = 2i + 4j determine the magnitude of their resultant.

(b) Camilla walks 5 km northeast, then 3 km due east and afterwards 2 km due south. Represent these displacements together with the resultant displacement graphically using the scale 1 unit = 1 km.

(c) Show that triangle ABC is right-angled where A = (-2,-1), B = (2,1) and C = (1,3).

(a) Resultant = a + b + c

$= (4i + 6j) + (4i + 10j) + (2i + 4j)$

$= (4 + 4 + 2)i + (6 + 10 + 4)j = 10i + 20j$

Magnitude = $\sqrt{(10^2 + 20^2)} = \sqrt{(100 + 400)} = \sqrt{500} = 10\sqrt{5} \approx 22.36$

Answer: 22.4

(b)

Northeast: 45°, vector $(5/\sqrt{2}, 5/\sqrt{2})$

East: (3, 0)

South: (0, -2)

Resultant: $(5/\sqrt{2} + 3, 5/\sqrt{2} - 2)$

Plot vectors head-to-tail, resultant from origin to final point.

(c) Vectors:

AB = (2 - (-2), 1 - (-1)) = (4, 2)

BC = (1 - 2, 3 - 1) = (-1, 2)

CA = (-2 - 1, -1 - 3) = (-3, -4)

Dot product AB · BC = 4(-1) + 2(2) = -4 + 4 = 0

Since AB · BC = 0, angle at B is 90°

Answer: Right-angled at B

5. (a) In the figure below, AB = 10 cm, AX = 6 cm, CX = 8 cm and AB is parallel to DC.

(i) Show whether triangles AXB and CXD are similar or not.

(ii) Find the length of CD.

(iii) Find the ratio of the areas of triangles AXB and CXD.

(b) Using a ruler and compass, construct an angle of 90°.

(a)(i) In triangles AXB and CXD:

∠AXB = ∠CXD (vertically opposite)

AB || DC, so ∠XAB = ∠XCD (alternate angles)

Triangles AXB and CXD are similar by AA

Answer: Similar by AA

(a)(ii) AXB ~ CXD, so AX/CX = AB/CD

6/8 = 10/CD

CD = 10 × 8/6 = 40/3 ≈ 13.33 cm

Answer: 40/3 cm

(a)(iii) Area ratio = (linear ratio)² = (AX/CX)² = (6/8)² = 9/16

Answer: 9:16

(b)

Draw line AB

Mark point O on AB

With compass at O, draw arcs intersecting at P above line

From P, draw arcs to intersect at Q

Line PQ ⊥ AB, forming 90°

Answer: Constructed as described

6. (a) In the preparation of fanta orange drink, a bottling filling machine can fill 1,500 bottles in 45 minutes. How many bottles will it fill in 4½ hours?

(b) If X varies directly as Y and inversely as W, find the values of a and b in the table below.

X	8	6	b
Y	4	a	2
W	2	3	4

(a) Rate = 1500 / 45 = 100/3 bottles/min

4½ hours = 4.5 × 60 = 270 minutes

Bottles = (100/3) × 270 = 9000

Answer: 9000 bottles

(b) X = kY/W

For X = 8, Y = 4, W = 2:

8 = k(4)/2

k = 4

X = 4Y/W

For X = 6, W = 3:

6 = 4(a)/3

a = 9/2

For Y = 2, W = 4:

b = 4(2)/4 = 2

Answer: a = 9/2, b = 2

7. A computer is advertised in a shop as having a list price of sh. 2,500,000 plus value added tax (VAT) of 20%. The sales manager offers a discount of 25% before adding the VAT. Calculate:

(a) The list price including VAT.

(b) The amount of discount before VAT is added.

(c) The reduced final price of the computer.

(a) List price = 2,500,000

VAT = 20% = 0.2 × 2,500,000 = 500,000

Price with VAT = 2,500,000 + 500,000 = 3,000,000

Answer: 3,000,000 shillings

(b) Discount = 25% = 0.25 × 2,500,000 = 625,000

Answer: 625,000 shillings

(c) Price after discount = 2,500,000 - 625,000 = 1,875,000

VAT = 0.2 × 1,875,000 = 375,000

Final price = 1,875,000 + 375,000 = 2,250,000

Answer: 2,250,000 shillings

8. (a) If the sum of n terms of a geometric progression with first term 1 and common ratio ½ is 31/16, find the number of terms.

(b) How many integers are there between 14 and 1,000 which are divisible by 17?

(a) GP: a = 1, r = 1/2

$S_n = a(1 - r^n)/(1 - r) = 1(1 - (1/2)^n)/(1 - 1/2) = 2(1 - (1/2)^n)$

$2(1 - (1/2)^n) = 31/16$

$1 - (1/2)^n = 31/32$

$(1/2)^n = 1/32$

$2^n = 32 = 2^5$

$n = 5$

Answer: 5 terms

(b) Multiples of 17: 17, 34, …, 986

$a_n = 17n \leq 1000$

$986 = 17 \times 58$

Numbers from 17 to 986: 58 terms

Answer: 58 integers

9. In the figure below, AE = 20 m, EB = $20\sqrt{2}$ m and DAE = 45°.

Find:

(a) The length: DE, AD and AB.

(b) The area of triangle ABE, leaving the answer in surd form.

(a) [No figure; assume triangle ABE with ∠DAE = 45°]

Assume right triangle at E, ∠AEB = 90°

AE = 20 m, EB = $20\sqrt{2}$ m, ∠DAE = 45°

In triangle ABE:

$AB^2 = AE^2 + EB^2 = 20^2 + (20\sqrt{2})^2 = 400 + 800 = 1200$

$AB = \sqrt{1200} = 20\sqrt{3}$ m

DE, AD: Need figure; assume D on extension, cannot solve without clarity

Answer: AB = $20\sqrt{3}$ m, DE and AD cannot be determined

(b) Area of triangle ABE = (1/2) × AE × EB

$= (1/2) \times 20 \times 20\sqrt{2} = 200\sqrt{2}$

Answer: $200\sqrt{2}$ m²

10. (a) Solve the equation $4x^2 - 32x + 12 = 0$ by using the quadratic formula.

(b) Anna is 6 years younger than her brother Jerry. If the product of their ages is 135, find how old is Anna and Jerry.

(a) $4x^2 - 32x + 12 = 0$

Divide by 4: $x^2 - 8x + 3 = 0$

$a = 1, b = -8, c = 3$

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

$= [8 \pm \sqrt{(64 - 12)}] / 2 = [8 \pm \sqrt{52}] / 2 = [8 \pm 2\sqrt{13}] / 2 = 4 \pm \sqrt{13}$

Answer: $x = 4 \pm \sqrt{13}$

(b) Anna = x, Jerry = x + 6

$x(x + 6) = 135$

$x^2 + 6x - 135 = 0$

$x = [-6 \pm \sqrt{(36 + 540)}] / 2 = [-6 \pm \sqrt{576}] / 2 = [-6 \pm 24] / 2$

x = 9 or x = -15

Anna = 9, Jerry = 15

Answer: Anna = 9, Jerry = 15

11. Zelda wants to buy oranges and mangoes for her children. The oranges are sold at sh. 150 each and mangoes at sh. 200 each. She must buy at least two of each kind of fruit but her shopping bag cannot hold more than 10 fruits. If the owner of the shop makes a profit of sh. 40 on each orange and sh. 60 on each mango, determine how many fruits of each kind Zelda must buy for the shop owner to realise maximum profit.

Let x = oranges, y = mangoes

Maximize: $P = 40x + 60y$

Constraints:

$x \geq 2, y \geq 2$

$x + y \leq 10$

Vertices:

(2,2): P = 40(2) + 60(2) = 80 + 120 = 200

(2,8): P = 40(2) + 60(8) = 80 + 480 = 560

(8,2): P = 40(8) + 60(2) = 320 + 120 = 440

Maximum at (2,8)

Answer: 2 oranges, 8 mangoes

12. The heights of 50 plants recorded by a certain researcher are given below:

56, 82, 70, 69, 72, 37, 28, 96, 52, 88, 41, 42, 50, 40, 51, 56, 48, 79, 29, 30, 66, 90, 99, 49, 77, 66, 61, 64, 97, 84, 72, 43, 73, 76, 76, 22, 46, 49, 48, 53, 98, 45, 87, 88, 27, 48, 80, 73, 54, 79

(a) Copy and complete this tally table for the data given above.

Height (cm)	Tally	Frequency
21-30		
31-40		
41-50		
51-60		
61-70		
71-80		
81-90		
91-100		

Use this table to:

(b) Draw a histogram for the height of the plants.

(c) Find the mean height of the plants (do not use the assumed mean method).

(d) Find the median of the heights of the plants.

(a) Tally table:

Height (cm)	Tally	Frequency
21-30		5
31-40		2
41-50		11
51-60		6
61-70		6
71-80		10
81-90		5
91-100		5

(b)

Histogram: Bars at 21-30 (5), 31-40 (2), ..., 91-100 (5), no gaps.

(c) Midpoints: 25.5, 35.5, 45.5, 55.5, 65.5, 75.5, 85.5, 95.5

Sum = (25.5×5) + (35.5×2) + (45.5×11) + (55.5×6) + (65.5×6) + (75.5×10) + (85.5×5) + (95.5×5)

= 127.5 + 71 + 500.5 + 333 + 393 + 755 + 427.5 + 477.5 = 3085

Mean = 3085 / 50 = 61.7

Answer: 61.7 cm

(d) Cumulative frequency:

21-30: 5

31-40: 7

41-50: 18

51-60: 24

61-70: 30

71-80: 40

81-90: 45

91-100: 50

Median: 50/2 = 25th term, in 61-70

Median ≈ 61 + (25 - 24)/6 × 10 ≈ 61.67

Answer: 61.7 cm

13. In the figure below, BC is a diameter of the circle, O is the centre of the circle and side CD of the cyclic quadrilateral ABCD is produced to E.

(a) With reasons, name the right angles in this figure.

(b) Show that ∠DAE = ∠ABC.

(c) If ∠DAE = 60° and ∠CAD = 25°, find:

(i) the value of ∠ABD,

(ii) the lengths AB and BD given that CB = 10 cm.

(a) BC is diameter, so ∠BAC = 90°, ∠BDC = 90° (angle in semicircle)

Answer: ∠BAC, ∠BDC; reason: angle in semicircle

(b) In cyclic quadrilateral ABCD, ∠DAE = ∠ABC (opposite angles of cyclic quadrilateral)

Answer: ∠DAE = ∠ABC by cyclic quadrilateral property

(c)(i) ∠DAE = ∠ABC = 60°

In triangle ABD, ∠CAD = ∠BAD = 25°

∠ABD = 180° - 60° - 25° = 95°

Answer: 95°

(c)(ii) use Pythagoras, $BC^2 = AB^2 + AC^2$

14. (a) What is a trial balance and what is its main purpose.

(b) On January 1st 2015 Semolina Women Group started a business with a capital in cash of 2,000,000/=

January 2 Purchased goods for cash 1,400,000/=

3 Sold goods for cash 1,000,000/=

6 Purchased goods for cash 600,000/=

15 Paid rent for cash 220,000/=

26 Paid wages for cash 220,000/=

15 Sold goods for cash 620,000/=

Prepare:

(i) The cash account and balance it.

(ii) The Trial Balance.

(a) Trial balance: List of all ledger account balances to check debit = credit

Purpose: Ensure arithmetic accuracy of accounts

Answer: Trial balance checks ledger accuracy

(b)(i) Cash Account:

Dr:

01/01 Capital: 2,000,000

03/01 Sales: 1,000,000

15/01 Sales: 620,000

Total: 3,620,000

Cr:

02/01 Goods: 1,400,000

06/01 Goods: 600,000

15/01 Rent: 220,000

26/01 Wages: 220,000

Total: 2,440,000

Balance c/d: 1,180,000

(b)(ii) Trial Balance:

| Account | Dr | Cr |

| Capital | | 2,000,000 |

| Goods (Purchases) | 2,000,000 | |

| Sales | | 1,620,000 |

| Rent | 220,000 | |

| Wages | 220,000 | |

| Cash | 1,180,000 | |

| Total | 3,620,000 | 3,620,000 |

15. (a) Find the inverse and identity matrix of A = (6 4; -2 5).

(b) Triangle OAB has vertices at O(0,0), A(2,1) and B(-1,3). If the triangle is enlarged by E = (2 0; 0 2) and then translated by T = (-3; -5), find the vertices of the triangle.

(c) Draw on the same x-y plane triangle OAB and the images after being:

(i) enlarged

(ii) translated

(a) A = (6 4; -2 5)

Det(A) = 6×5 - 4×(-2) = 30 + 8 = 38

Inverse: (1/38)(5 -4; 2 6) = (5/38 -2/19; 1/19 3/19)

Identity: I = (1 0; 0 1)

Answer: Inverse = (5/38 -2/19; 1/19 3/19), I = (1 0; 0 1)

(b) Enlarge by E:

O(0,0) → (0,0)

A(2,1) → (4,2)

B(-1,3) → (-2,6)

Translate by (-3,-5):

(0,0) → (-3,-5)

(4,2) → (1,-3)

(-2,6) → (-5,1)

Answer: (-3,-5), (1,-3), (-5,1)

(c)

OAB: O(0,0), A(2,1), B(-1,3)

Enlarged: O'(0,0), A'(4,2), B'(-2,6)

Translated: O''(-3,-5), A''(1,-3), B''(-5,1)

Answer: As described

16. (a) A function f is defined on the set of integers as follows:

$f(x) = \{ 1+x \text{ if } 1 \leq x \leq 2; 2x-1 \text{ if } 2 \leq x \leq 4; 3x-10 \text{ if } 4 \leq x \leq 6 \}$

(i) Draw a pictorial diagram for f(x).

(ii) Find the domain and range of f(x).

(b) Given that $f(x) = (5x + 7)/(x + 2)$, find $f^{-1}(4)$.

(c) In a yard there are 500 vehicles, of which 160 are cars, 130 are vans and the remaining are lorries. If every vehicle has an equal chance to leave, find the probability of:

(i) A van leaving first,

(ii) A lorry leaving first,

(iii) A car leaving second if either a lorry or van had left first.

(a)(i) [Cannot draw]

x = 1: f(1) = 2

x = 2: f(2) = 3 or 3

x = 3: f(3) = 5

x = 4: f(4) = 7 or 7

x = 5: f(5) = 5

x = 6: f(6) = 8

Arrows: 1→2, 2→3, 3→5, 4→7, 5→5, 6→8

(a)(ii) Domain: {1, 2, 3, 4, 5, 6}

Range: {2, 3, 5, 7, 8}

Answer: Domain: {1,2,3,4,5,6}, Range: {2,3,5,7,8}

(b) $y = (5x + 7)/(x + 2)$

$y(x + 2) = 5x + 7$

$yx + 2y = 5x + 7$

$yx - 5x = 7 - 2y$

$x(y - 5) = 7 - 2y$

$x = (7 - 2y)/(y - 5)$

$f^{-1}(y) = (7 - 2y)/(y - 5)$

$f^{-1}(4) = (7 - 8)/(4 - 5) = -1/-1 = 1$

Answer: 1

(c) Total = 500, Cars = 160, Vans = 130, Lorries = 500 - 160 - 130 = 210

(i) P(van) = 130/500 = 13/50

Answer: 13/50

(ii) P(lorry) = 210/500 = 21/50

Answer: 21/50

(iii) First is lorry or van: 210 + 130 = 340

Remaining: 499 vehicles, 160 cars

P(car second) = 160/499

Answer: 160/499