THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2019**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Mangoes are to be exactly divided into groups of 20, 30 or 36. What is the minimum number of mangoes required?

(b) Mary was given 60,000 shillings by her mother. She spent 35 percent of the money to buy shoes and 10 percent of the remaining money to buy books. How much money remained?

(a) Minimum number = LCM of 20, 30, 36

$20 = 2^2 \times 5$

$30 = 2 \times 3 \times 5$

$36 = 2^2 \times 3^2$

$LCM = 2^2 \times 3^2 \times 5 = 180$

Answer: 180 mangoes

(b) Shoes = 35% of 60,000 = $0.35 \times 60,000 = 21,000$

Remaining = 60,000 - 21,000 = 39,000

Books = 10% of 39,000 = $0.1 \times 39,000 = 3,900$

Remaining = 39,000 - 3,900 = 35,100

Answer: 35,100 shillings

2. (a) Find the value of x if √(5^(2x-5)) - 9 = 116.

(b) Find the value of the expression $(3.143 \times (0.81)^2) / \sqrt{35}$ by using mathematical tables.

(a) √(5^(2x-5)) - 9 = 116

√(5^(2x-5)) = 125

5^(2x-5) = $125^2 = 15625$

$125 = 5^3$, so $15625 = 5^6$

5^(2x-5) = 5^6

2x - 5 = 6

2x = 11

x = 11/2

Answer: $x = 11/2$

(b) Expression $= (3.143 \times 0.81^2) / \sqrt{35}$

$\log((3.143 \times 0.81^2) / \sqrt{35}) = \log(3.143) + 2\log(0.81) - (1/2)\log(35)$

$\log(3.143) \approx 0.4973$

$\log(0.81) = \log(81/100) = \log(3^4/10^2) = 4\log 3 - 2 \approx 4 \times 0.4771 - 2 = -0.0916$

$\log(35) \approx \log(5 \times 7) \approx 0.6990 + 0.8451 = 1.5441$

Numerator: $0.4973 + 2 \times (-0.0916) = 0.4973 - 0.1832 = 0.3141$

Denominator: $(1/2) \times 1.5441 = 0.77205$

log = $0.3141 - 0.77205 = -0.45795$

Antilog$(-0.45795) \approx 0.348$

Answer: 0.348

3. (a) Use the following Venn diagram to answer the questions that follow.

(i) Find the number of subsets of set B'.

(ii) Find the elements of set $A' \cap B$.

(iii) If an element is picked at random from the universal set (U), find the probability that it is not an element of set B.

(b) The Ministry of Business and Industries has planned to employ 54 people who will work in the business sector, 36 people who will work in industries sector only, 12 people who will work in both sectors and 21 people who will neither work in business sector nor in industries sector. How many people will be employed by the Ministry? (Use a Venn diagram).

(a) [No Venn diagram; assume $U = \{1,2,3,4,5,6,7,8\}$, $A = \{1,3,5,7\}$, $B = \{2,3,5,6\}$]

(i) $B' = \{1,4,7,8\}$, $n(B') = 4$

Number of subsets $= 2^4 = 16$

Answer: 16

(ii) $A' = \{2,4,6,8\}$, $B = \{2,3,5,6\}$

$A' \cap B = \{2,6\}$

Answer: {2,6}

(iii) n(U) = 8, n(B) = 4

P(not B) = (8 - 4)/8 = 1/2

Answer: 1/2

(b) Business (B): 54 - 12 = 42 (only B)

Industries (I): 36 (only I)

B ∩ I: 12

Neither: 21

Total = 42 + 36 + 12 + 21 = 111

Answer: 111 people

4. (a) Find the equation of a line which passes through the point A(-3,4) and which is parallel to the line 3x + 4y - 15 = 0.

(b) The points P, Q and R are (5,-3), (-6,1) and (1,8) respectively. Show that these points form an isosceles triangle.

(a) 3x + 4y - 15 = 0

4y = -3x + 15

y = (-3/4)x + 15/4

Slope = -3/4

Parallel line through (-3,4):

y - 4 = (-3/4)(x + 3)

y - 4 = (-3/4)x - 9/4

y = (-3/4)x + 7/4

Answer: y = (-3/4)x + 7/4

(b) Distances:

PQ = $\sqrt{((5 - (-6))^2 + (-3 - 1)^2)} = \sqrt{(11^2 + (-4)^2)} = \sqrt{137}$

QR = $\sqrt{((-6 - 1)^2 + (1 - 8)^2)} = \sqrt{((-7)^2 + (-7)^2)} = \sqrt{98} = 7\sqrt{2}$

PR = $\sqrt{((5 - 1)^2 + (-3 - 8)^2)} = \sqrt{(4^2 + (-11)^2)} = \sqrt{137}$

PQ = PR, so isosceles

Answer: Isosceles (PQ = PR)

5. (a) If AB is parallel to CD and CD is a transversal, sketch the line segments and label on the same diagram the following pairs of angles:

(i) corresponding angles of a and g.

(ii) alternate interior angles of f and g.

(iii) vertically opposite angles c and d.

(b) Given that triangle ABC is similar to triangle PQR, AB = 4 cm, BC = 5 cm, PQ = 18 cm and angle PQR is 30°, find the area of triangle PQR.

(a) [Cannot sketch; describe]

AB || CD, CD transversal

Angles: a, b (at A), c, d (at B), e, f (at C), g, h (at D)

(i) Corresponding: a and g (same side of transversal, above/below lines)

(ii) Alternate interior: f and g (opposite sides of transversal, inside lines)

(iii) Vertically opposite: c and d (at intersection B)

Answer: As described

(b) ABC ~ PQR

PQ/AB = 18/4 = 4.5

Area ratio = $(4.5)^2 = 20.25$

Area ABC = $(1/2) \times 4 \times 5 \times \sin(30°) = (1/2) \times 20 \times (1/2) = 5$ cm²

Area PQR = $5 \times 20.25 = 101.25$ cm²

Answer: 101.25 cm²

6. (a) Mr. Ogango from Kenya visited Tanzania. He had 5,000 Kenya shillings (Kshs) and wanted to change the money into US dollars. If 1 US dollar was equivalent to 2500 Tanzania shillings (Tshs) and Ksh 1 was equivalent to Tshs 20, how much US dollars did he get?

(b) A gardener has found the time t to cut the grass on a square field varies directly as the square of its length (L) and inversely as a number of men (m) doing that job. If 5 men cut grass on a field of side 50 m in 3 hours, how many more men are required to cut grass on a field of side 100 m in 5 hours? Assume that the men are working on the same pace.

(a) 5000 Kshs = 5000 × 20 = 100,000 Tshs

US dollars = 100,000 / 2500 = 40

Answer: 40 US dollars

(b) $t = k L^2 / m$

$3 = k (50^2) / 5$

$3 = k \times 2500 / 5$

$k = 3 / 500 = 0.006$

For L = 100, t = 5:

$5 = 0.006 \times 100^2 / m$

$5 = 0.006 \times 10000 / m$

$m = 60 / 5 = 12$

Additional men = 12 - 5 = 7

Answer: 7 men

7. (a) Misumbwi, Shuma and Kiyando contributed 770,000, 560,000 and 1,050,000 shillings respectively to start a business. Find the ratio of their contribution in its simplest form.

(b) Use the following trial balance to prepare trading, profit and loss account of Mr. Rwaichi as at 31st December 2015.

TRIAL BALANCE AS AT 31ST DECEMBER, 2015

Account name	Dr	Cr
Cash	1750000	
Capital		2300000

Purchases	2300000	
Rent	200000	
Furniture	550000	
Shelves	350000	
Sales		2500000
Salary	250000	
Wages	100000	
Total	5500000	5500000

(a) 770,000 : 560,000 : 1,050,000

Divide by 10,000: 77 : 56 : 105

GCD = 7

77/7 : 56/7 : 105/7 = 11 : 8 : 15

Answer: 11:8:15

(b) Trading Account:

Dr:

Purchases: 2,300,000

Cr:

Sales: 2,500,000

Gross profit: 2,500,000 - 2,300,000 = 200,000

Profit and Loss Account:

Dr:

Rent: 200,000

Salary: 250,000

Wages: 100,000

Total: 550,000

Net loss: 550,000 - 200,000 = 350,000

Cr:

Gross profit: 200,000

Answer: Gross profit: 200,000; Net loss: 350,000

8. (a) Given that 49, x and 81 are consecutive terms of a geometric progression. Find:

(i) the value of x.

(ii) the geometric mean.

(b) A wall is in the shape of a trapezium. The first level of the wall is made up of 50 bricks whereas the top level has 14 bricks. If the levels differ from each other by 4 bricks, determine the number of:

(i) levels of the bricks.

(ii) bricks used to make the wall.

(a)(i) $x/49 = 81/x$

$x^2 = 49 \times 81 = 3969$

$x = \sqrt{3969} = 63$

Answer: x = 63

(a)(ii) Geometric mean = $\sqrt{(49 \times 81)} = \sqrt{3969} = 63$

Answer: 63

(b)(i) AP: $a_1 = 50$, a_n = 14, d = -4

$a_n = a_1 + (n-1)d$

$14 = 50 + (n-1)(-4)$

$14 = 50 - 4n + 4$

$4n = 40$

$n = 10$

Answer: 10 levels

(b)(ii) $S_n = n/2 (a_1 + a_n)$

$S_{10} = 10/2 (50 + 14) = 5 \times 64 = 320$

Answer: 320 bricks

9. (a) The following diagram shows the location of the houses A, B and C. How far is house A from house B? Give the answer to the nearest metre.

(b) A rectangular frame is made of wooden bars. The diagonal of the frame is 25 cm long and its width is 15 cm. Find the length of the frame.

(a) [No diagram; cannot solve]

Answer: Cannot solve without diagram

(b) Diagonal = 25 cm, width = 15 cm

$\text{Length}^2 + 15^2 = 25^2$

$\text{Length}^2 + 225 = 625$

$\text{Length}^2 = 400$

Length = 20 cm

Answer: 20 cm

10. (a) Factorise the quadratic expression $3x^2 - 11x - 20$ by splitting the middle term.

(b) Solve the equation $2x^2 - 3x - 5 = 0$ by completing the square.

(a) $3x^2 - 11x - 20$

Find p, q: $p \times q = 3 \times (-20) = -60$, $p + q = -11$

$p = -15$, $q = 4$

$3x^2 - 15x + 4x - 20$

$= (3x^2 - 15x) + (4x - 20)$

$= 3x(x - 5) + 4(x - 5)$

$= (3x + 4)(x - 5)$

Answer: $(3x + 4)(x - 5)$

(b) $2x^2 - 3x - 5 = 0$

$x^2 - (3/2)x - 5/2 = 0$

$x^2 - (3/2)x = 5/2$

$(x - 3/4)^2 - (3/4)^2 = 5/2$

$(x - 3/4)^2 - 9/16 = 40/16$

$(x - 3/4)^2 = 49/16$

$x - 3/4 = \pm 7/4$

$x = 10/4 = 5/2$ or $x = -4/4 = -1$

Answer: $x = 5/2, -1$

11. The number of patients who attended maternity clinic daily in June 2017 in a certain village was recorded as follows:

52, 61, 42, 27, 38, 44, 56, 36, 73, 22, 41, 48, 77, 30, 46, 43, 72, 63, 43, 76, 47, 53, 38, 55, 60, 51, 47, 58, 33, 37

(a) Make a frequency distribution by grouping the number of patients in the class intervals: 20-29, 30-39, 40-49, …

(b) By using the frequency distribution table obtained in part (a), calculate the mean number of patients per day.

(c) Construct a pie chart for the frequency distribution obtained in part (a).

(a) 20-29: 22, 27 → 2

30-39: 30, 33, 36, 37, 38, 38 → 6

40-49: 41, 42, 43, 43, 44, 46, 47, 47, 48 → 9

50-59: 51, 52, 53, 55, 56, 58 → 6

60-69: 60, 61, 63 → 3

70-79: 72, 73, 76, 77 → 4

Table:

| Class | Frequency |

| 20-29 | 2 |

| 30-39 | 6 |

| 40-49 | 9 |

| 50-59 | 6 |

| 60-69 | 3 |

| 70-79 | 4 |

(b) Midpoints: 24.5, 34.5, 44.5, 54.5, 64.5, 74.5

Sum = (24.5×2) + (34.5×6) + (44.5×9) + (54.5×6) + (64.5×3) + (74.5×4)

= 49 + 207 + 400.5 + 327 + 193.5 + 298 = 1475

Mean = 1475 / 30 ≈ 49.167

Answer: 49.2 patients

(c) Total = 30

Angles:

20-29: (2/30) × 360° = 24°

30-39: (6/30) × 360° = 72°

40-49: (9/30) × 360° = 108°

50-59: (6/30) × 360° = 72°

60-69: (3/30) × 360° = 36°

70-79: (4/30) × 360° = 48°

Pie chart with sectors labeled as above

12. (a) A ship sails from Pemba (4.5°S, 39.5°E) to Dar es Salaam (7.5°S, 39.5°E). If it leaves Pemba at 11:30 am and arrived in Dar es Salaam at 13:30 pm, use $\pi = 22/7$ and $R_E = 6370$ km to find the speed of the ship in km/h.

(b) Sketch a square pyramid whose base is PQRS, vertex is at W and centre is at N, then answer the questions that follow:

(i) State the projection of RW on PQRS.

(ii) Name the angle between WS and the plane PQRS.

(c) The volume of a square pyramid is 28.2 cm³. If the sides of its base are 4 cm long, find the height of the pyramid correct to one decimal place.

(a) Latitude difference = 7.5° - 4.5° = 3°

$\theta = 3 \times \pi/180 = 3 \times (22/7) / 180 = 11/210$ radians

Distance = $6370 \times 11/210 \approx 333.524$ km

Time = 13:30 - 11:30 = 2 hours

Speed = $333.524 / 2 \approx 166.762$

Answer: 166.8 km/h

(b) [Cannot sketch; describe]

Square base PQRS, vertex W, center N

(i) Projection of RW on PQRS: Point R

(ii) Angle between WS and plane PQRS: ∠WSN

Answer: (i) R, (ii) ∠WSN

(c) Volume = (1/3) × base area × height

Base area = $4 \times 4 = 16$ cm²

$28.2 = (1/3) \times 16 \times h$

$h = 28.2 \times 3 / 16 = 5.2875$

Answer: 5.3 cm

13. (a) (i) Given the matrices P = (2 -3; 5 4) and Q = (9 12; -15 3). Find 2P - (1/3)Q.

(ii) If the matrix (4k 8; 2 9k) is singular, find the possible values of k.

(b) Solve the following system of linear equations by using the inverse matrix method:

{2x + 3y = 7, y = (1/2)x}

(c) By using the transformation matrix (1 0; 0 -1), find the image of the point A(-2,3). Hence, state the axis in which the point is reflected.

(a)(i) 2P = 2(2 -3; 5 4) = (4 -6; 10 8)

(1/3)Q = (1/3)(9 12; -15 3) = (3 4; -5 1)

2P - (1/3)Q = (4 - 3, -6 - 4; 10 - (-5), 8 - 1) = (1 -10; 15 7)

Answer: (1 -10; 15 7)

(a)(ii) Singular: det = 0

det = 4k × 9k - 8 × 2 = 36k² - 16 = 0

36k² = 16

k² = 4/9

k = ±2/3

Answer: k = 2/3, -2/3

(b) y = (1/2)x

2x + 3((1/2)x) = 7

2x + (3/2)x = 7

(7/2)x = 7

x = 2

y = (1/2) × 2 = 1

Matrix form: (2 3; -1/2 1)(x; y) = (7; 0)

A = (2 3; -1/2 1)

det(A) = 2 × 1 - 3 × (-1/2) = 2 + 3/2 = 7/2

Inverse = (2/7)(1 -3; 1/2 2)

(x; y) = (2/7)(1 -3; 1/2 2)(7; 0) = (2/7)(7; 7/2) = (2; 1)

Answer: x = 2, y = 1

(c) (1 0; 0 -1)(-2; 3) = (-2; -3)

Reflection in y-axis (x unchanged, y negated)

Answer: (-2, -3), y-axis

14. (a) Calculate the values of f(1) and f(-π) if f is defined by

f(x) = {x + 2 for x < 0; 2 for 0 ≤ x ≤ 2}

(b) Using the information given in part (a), find $f^{-1}(-1)$.

(c) A trader has a space for 5 refrigerators. The trader plans to spend 2,400,000 shillings to buy refrigerators of two brands, Hitachi and Sony. Each Hitachi refrigerator costs 600,000 shillings whereas each Sony refrigerator costs 400,000 shillings. The unit profits for Hitachi and Sony refrigerators are 200,000 shillings and 150,000 shillings respectively. Denoting x and y as the number of Hitachi and Sony refrigerators respectively, determine the number of refrigerators for each brand that maximizes profit.

(a) f(1): $0 \leq 1 \leq 2$, f(1) = 2

f(-π): $-\pi < 0$, $f(-\pi) = -\pi + 2$

Answer: f(1) = 2, $f(-\pi) = 2 - \pi$

(b) f(x) = x + 2 for x < 0, range: y > 2

f(x) = 2 for $0 \leq x \leq 2$, range: y = 2

$f^{-1}(-1)$: No x such that f(x) = -1

(c) Maximize: P = 200,000x + 150,000y

Constraints:

$x + y \leq 5$

$600{,}000x + 400{,}000y \leq 2{,}400{,}000 \rightarrow 3x + 2y \leq 12$

$x \geq 0, y \geq 0$

Vertices:

(0,0): P = 0

(0,5): P = 150,000 × 5 = 750,000

(4,0): P = 200,000 × 4 = 800,000

(2,3): 3x + 2y = 12, x + y = 5 → x = 2, y = 3

P = 200,000 × 2 + 150,000 × 3 = 400,000 + 450,000 = 850,000

Maximum at (2,3)

Answer: 2 Hitachi, 3 Sony