THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2020**

Instructions

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

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1. (a) Simplify the expression $(0.25 \times 8.85 \times 25) / (0.00625 \times 0.5)$ without using mathematical tables, expressing the answer correct to two significant figures.

(b) (i) Mr. Magani set an examination weighing a total of 96 marks with the following distributions: 20% of the marks were awarded for reading, 40% for writing, 15% for practical and the remaining percentage for spelling. Find the marks that were awarded for spelling.

(ii) Three airplanes arrived at Kilimanjaro International Airport (KIA) at the intervals of 30 minutes, 40 minutes and 55 minutes. If all three airplanes arrived at KIA at 2:00 p.m. on Saturday, when and at what time would they arrive together again?

(a) $(0.25 \times 8.85 \times 25) / (0.00625 \times 0.5)$

Numerator: $0.25 = 1/4$, $8.85 = 885/100$, 25

$= (1/4) \times (885/100) \times 25 = (885 \times 25) / (4 \times 100) = 22125 / 400$

Denominator: $0.00625 = 1/160$, $0.5 = 1/2$

$= (1/160) \times (1/2) = 1/320$

Expression: $(22125 / 400) / (1/320) = (22125 / 400) \times 320$

$= (22125 \times 320) / 400 = 22125 \times 0.8 = 17700$

To 2 significant figures: 18000

Answer: 18000

(b)(i) Total = 96 marks

Reading: $20\% = 0.2 \times 96 = 19.2$

Writing: $40\% = 0.4 \times 96 = 38.4$

Practical: $15\% = 0.15 \times 96 = 14.4$

Used $= 19.2 + 38.4 + 14.4 = 72$

Spelling $= 96 - 72 = 24$

(b)(ii) Intervals: 30, 40, 55 minutes

LCM $= 30 = 2 \times 3 \times 5$, $40 = 2^3 \times 5$, $55 = 5 \times 11$

LCM $= 2^3 \times 3 \times 5 \times 11 = 1320$ minutes

$1320 / 60 = 22$ hours

2:00 p.m. Saturday + 22 hours = 12:00 p.m. Sunday

2. (a) If $(\sqrt{3}) / (2 + \sqrt{3}) = a + b\sqrt{c}$, find the values of a, b and c.

(b) (i) Solve the equation $(9 / \sqrt{3})^{(2x)} = 1/81$.

(ii) Given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the value of $\log (21 / 4)$ without using mathematical table.

(a) $(\sqrt{3}) / (2 + \sqrt{3})$

Rationalize: $(\sqrt{3}) / (2 + \sqrt{3}) \times (2 - \sqrt{3}) / (2 - \sqrt{3})$

$= \sqrt{3}(2 - \sqrt{3}) / (4 - 3) = 2\sqrt{3} - 3$

$= -3 + 2\sqrt{3}$

$a = -3, b = 2, c = 3$

Answer: $a = -3, b = 2, c = 3$

(b)(i) $(9 / \sqrt{3})^{(2x)} = 1/81$

$9 / \sqrt{3} = 9 / 3^{(1/2)} = 3^{(2 - 1/2)} = 3^{(3/2)}$

$1/81 = 81^{(-1)} = (3^4)^{(-1)} = 3^{(-4)}$

$(3^{(3/2)})^{(2x)} = 3^{(-4)}$

$3^{(3x)} = 3^{(-4)}$

$3x = -4$

$x = -4/3$

Answer: $x = -4/3$

(b)(ii) $\log (21 / 4) = \log 21 - \log 4$

$\log 21 = \log (3 \times 7) = \log 3 + \log 7$

$\log 4 = \log (2^2) = 2 \log 2$

Assume $\log 7 \approx 0.8451$

$\log 21 = 0.4771 + 0.8451 = 1.3222$

$\log 4 = 2 \times 0.3010 = 0.6020$

log (21 / 4) = 1.3222 - 0.6020 = 0.7202

Answer: 0.7202

3. In a certain school, 40 students were asked about whether they like tennis or football or both. It was found that the number of students who like both tennis and football was three times the number of students who like tennis only. Furthermore, the number of students who like football only was 6 more than twice the number of students who like tennis only. However, 4 students like neither tennis nor football.

(a) Represent this information in a Venn diagram, letting x be the number of students who like tennis only.

(b) Use the results obtained in part (a) to determine the probability that a student selected at random likes;

(i) football only.

(ii) both football and tennis.

(a) Tennis only = x

Both = 3x

Football only = 2x + 6

Neither = 4

Total = x + 3x + (2x + 6) + 4 = 40

6x + 10 = 40

6x = 30

x = 5

Tennis only = 5, Both = 15, Football only = 16

Venn diagram: [Describe]

Tennis circle: 5 (only), 15 (both)

Football circle: 16 (only), 15 (both)

Outside: 4

(b)(i) P(football only) = 16 / 40 = 2/5

Answer: 2/5

(b)(ii) P(both) = 15 / 40 = 3/8

Answer: 3/8

4. (a) (i) A line whose gradient is 3/2 has the x-intercept of -3. Find the equation of the line in the form $y = mx + c$, where m and c are constants.

(ii) Find the length of a line segment joining the points (3,-2) and (15,3).

(b) A boat sails due North at a speed of 120 km/h and a wind blows at a speed of 40 km/h due East. Find the actual speed of the boat. Use $\sqrt{10} = 3.16$.

(a)(i) x-intercept = -3, point (-3, 0)

Gradient m = 3/2

$y = (3/2)(x + 3)$

$y = (3/2)x + 9/2$

Answer: $y = (3/2)x + 9/2$

(a)(ii) Distance = $\sqrt{(15 - 3)^2 + (3 - (-2))^2}$

$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

Answer: 13 units

(b) Boat: (0, 120), Wind: (40, 0)

Resultant: (40, 120)

Speed = $\sqrt{40^2 + 120^2} = \sqrt{1600 + 14400} = \sqrt{16000} = 40\sqrt{10}$

$= 40 \times 3.16 = 126.4$

Answer: 126.4 km/h

5. (a) In the following triangle ABC, AB = 8 cm, BC = 11.3 cm and ∠ABC = 30°. Find the area of the triangle.

(b) (i) Find the perimeter of a regular hexagon inscribed in a circle whose radius is 100 m.

(ii) Given that AB/KL = BT/LC = TA/CK = 3, where AB, BT and TA are the sides of the triangle ABT and KL, LC and CK are the sides of triangle KLC; what does this information imply?

(a) Area = $(1/2) \times AB \times BC \times \sin(\angle ABC)$

$= (1/2) \times 8 \times 11.3 \times \sin(30°) = (1/2) \times 8 \times 11.3 \times (1/2)$

$= 22.6$

Answer: 22.6 cm^2

(b)(i) Hexagon side = radius = 100 m

Perimeter = $6 \times 100 = 600$ m

Answer: 600 m

(b)(ii) AB/KL = BT/LC = TA/CK = 3

Triangles ABT and KLC are similar (side ratios equal)

Answer: Triangles ABT and KLC are similar

6. (a) The variables t and z in the following table are related by the formula $z = a t^n$ where a is a constant and n is a positive integer.

(i) Use the data from the table to determine the values of a and n.

(ii) Use the values of a and n obtained in part (a)(i) to complete the following table.

t	1	2	3	4	5
z	0.5	4	13.5		

(b) If v varies directly as the square of x and inversely as $\sqrt{y}$, given that v = 18 when x = 3 and y = 16, find the value of v when x = 5 and y = 4.

(a)(i) $z = a t^n$

$t = 1, z = 0.5$: $0.5 = a \times 1^n \rightarrow a = 0.5$

$t = 2, z = 4$: $4 = 0.5 \times 2^n$

$2^n = 8 = 2^3$

$n = 3$

$z = 0.5 t^3$

Answer: a = 0.5, n = 3

(a)(ii) $z = 0.5 t^3$

$t = 4$: $z = 0.5 \times 4^3 = 0.5 \times 64 = 32$

$t = 5$: $z = 0.5 \times 5^3 = 0.5 \times 125 = 62.5$

Table:

t	1	2	3	4	5
z	0.5	4	13.5	32	62.5

(b) $v = k x^2 / \sqrt{y}$

$18 = k (3^2) / \sqrt{16}$

$18 = k \times 9 / 4$

$k = 18 \times 4 / 9 = 8$

$v = 8 x^2 / \sqrt{y}$

For $x = 5$, $y = 4$:

$v = 8 \times 5^2 / \sqrt{4} = 8 \times 25 / 2 = 100$

7. (a) (i) A school has 2,000 students, of whom 1,500 are boys. What is the ratio of boys to girls in the school?

(ii) Matiku bought a book for Tshs. 120,000. A year later, he sold the book at a profit of 20%. What was the selling price of the book?

(b) Halima started a business on 1st September, 2018 with a capital of Tshs. 25,000/= in cash.

September 2, bought goods for cash 15,000/=

3, sold goods for cash 3,000/=

5, sold goods for cash 5,000/=

6, paid carriage on goods 500/=

9, sold goods for cash 14,000/=

15, bought goods for cash 1,000/=

19, paid rent 2,000/=

20, purchased goods 6,000/=

27, paid wages 5,000/=

28, sold goods on credit 1,000/=

By using these transactions, prepare the cash account.

(a)(i) Boys = 1500, Girls = 2000 - 1500 = 500

Ratio = 1500 : 500 = 3 : 1

Answer: 3:1

(a)(ii) Cost = 120,000

Selling price = 120,000 × 1.2 = 144,000

Answer: 144,000 Tshs

(b) Cash Account:

Dr:

01/09 Capital: 25,000

03/09 Sales: 3,000

05/09 Sales: 5,000

09/09 Sales: 14,000

Total: 47,000

Cr:

02/09 Goods: 15,000

06/09 Carriage: 500

15/09 Goods: 1,000

19/09 Rent: 2,000

20/09 Goods: 6,000

27/09 Wages: 5,000

Total: 29,500

Balance c/d: 17,500

8. (a) Find the first term and the common difference of an arithmetic progression whose 5th term is 21 and 8th term is 30.

(b) Find the 10th term of a sequence whose first three consecutive terms are 5, 15 and 45. (Leave the answer in exponent form.)

(a) 5th term: $a + 4d = 21$ (1)

8th term: $a + 7d = 30$ (2)

(2) - (1): $3d = 9$

$d = 3$

$a + 4(3) = 21$

$a = 21 - 12 = 9$

Answer: $a = 9$, $d = 3$

(b) Sequence: 5, 15, 45

$r = 15/5 = 3$

GP: $a = 5$, $r = 3$

$a_{10} = 5 \times 3^{(10-1)} = 5 \times 3^9$

Answer: 5×3^9

9. (a) In the following figure, AP is perpendicular to BC, AB = 13 cm, BP = 5 cm and AC = 15 cm.

Calculate the lengths of AP and CP.

(b) From the top of a building 75 m high, John sees a lorry and a minibus along the road, both being on one side of the building at the angles of depression of 30° and 60° respectively.

(i) Sketch a diagram representing this information.

(ii) Determine the distance between the cars, leaving the answer in surd form.

(a) In triangle APB, $\angle APB = 90°$

$AP^2 + BP^2 = AB^2$

$AP^2 + 5^2 = 13^2$

$AP^2 = 169 - 25 = 144$

$AP = 12$ cm

In triangle APC, $\angle APC = 90°$

$AP^2 + CP^2 = AC^2$

$12^2 + CP^2 = 15^2$

$CP^2 = 225 - 144 = 81$

$CP = 9$ cm

Answer: $AP = 12$ cm, $CP = 9$ cm

(b)(i)

Building height 75 m, point T at top

Lorry at L, minibus at M, on same side

$\angle LTM = 30°$ (lorry), $\angle MTM = 60°$ (minibus)

(b)(ii) $\tan 60° = 75 / d_1$

$d_1 = 75 / \sqrt{3}$

$\tan 30° = 75 / d_2$

$d_2 = 75 / (1/\sqrt{3}) = 75\sqrt{3}$

Distance $LM = d_2 - d_1 = 75\sqrt{3} - 75/\sqrt{3}$

$= 75 (\sqrt{3} - 1/\sqrt{3}) = 75 (3 - 1) / \sqrt{3} = 150 / \sqrt{3} = 50\sqrt{3}$

Answer: $50\sqrt{3}$ m

10. (a) Rachel is three years older than her brother John. Three years to come, the product of their ages will be 130 years. Formulate a quadratic equation representing this information. Hence, by using the quadratic formula, find their present ages.

(b) The sum of the squares of two consecutive positive numbers is 61. Find the numbers.

(a) John = x, Rachel = $x + 3$

In 3 years: John = $x + 3$, Rachel = $x + 6$

$(x + 3)(x + 6) = 130$

$x^2 + 9x + 18 = 130$

$x^2 + 9x - 112 = 0$

$x = [-9 \pm \sqrt{(81 + 448)}] / 2$

$= [-9 \pm \sqrt{529}] / 2 = [-9 \pm 23] / 2$

$x = 7$ or $x = -16$

John = 7, Rachel = 10

Answer: John = 7, Rachel = 10

(b) Numbers: n, n + 1

$n^2 + (n + 1)^2 = 61$

$n^2 + n^2 + 2n + 1 = 61$

$2n^2 + 2n - 60 = 0$

$n^2 + n - 30 = 0$

$n = [-1 \pm \sqrt{(1 + 120)}] / 2 = [-1 \pm 11] / 2$

$n = 5$ or $n = -6$

Positive: n = 5, n + 1 = 6

Answer: 5, 6

11. The following data represent the marks scored by 36 students of a certain school in Geography examination:

72, 76, 90, 89, 74, 82, 63, 74, 70, 73, 58, 71, 55, 62, 65, 74, 71, 64, 71, 85, 70, 61, 64, 75, 51, 83, 50, 61, 83, 68, 70, 80, 50, 60, 66, 68

(a) Prepare a frequency distribution table representing the given data by using the class intervals: 50-54, 55-59, 60-64, and so on.

(b) Use the frequency distribution table obtained in part (a) to:

(i) draw a histogram.

(ii) calculate the median. Write the answer correct to 2 decimal places.

(a) 50-54: 50, 51, 50 → 3

55-59: 55, 58 → 2

60-64: 60, 61, 61, 62, 63, 64, 64 → 7

65-69: 65, 66, 68, 68 → 4

70-74: 70, 70, 70, 71, 71, 71, 72, 73, 74, 74, 74 → 11

75-79: 75, 76 → 2

80-84: 80, 82, 83, 83 → 4

85-89: 85, 89 → 2

90-94: 90 → 1

Table:

Class	Frequency
50-54	3
55-59	2
60-64	7
65-69	4
70-74	11
75-79	2
80-84	4
85-89	2
90-94	1

(b)(i)

Histogram: Bars at 50-54 (3), 55-59 (2), ..., 90-94 (1)

(b)(ii) Cumulative frequency:

50-54: 3

55-59: 5

60-64: 12

65-69: 16

70-74: 27

75-79: 29

80-84: 33

85-89: 35

90-94: 36

Median: 36/2 = 18th term, in 70-74

Median ≈ 70 + (18 - 16)/11 × 5 ≈ 70.91

12. (a) Two towns, A and B, are located at (10°S, 38°E) and (10°S, 43°E) respectively.

(i) Find the distance between the two towns in kilometres. (Use radius of the Earth, R = 6400 km and π = 3.14). Give the answer to the nearest whole number.

(ii) Suppose a ship is sailing at 50 km/h from town A to town B. Using the answer obtained in part (a)(i), find how long will the ship take to reach town B.

(b) The following figure represents a rectangular prism in which PQ = 12 cm, QR = 8 cm and RY = 4 cm.

Find;

(i) The total surface area.

(ii) the angle between the planes PTZW and QRZW.

(c) Calculate the volume of a cone whose base radius is 12 cm and slant height is 20 cm. (Use π = 3.14).

(a)(i) Longitude difference = 43° - 38° = 5°

Distance = $R \times \theta \times \cos(10°)$

$\theta = 5 \times \pi/180 = 5 \times 3.14 / 180 \approx 0.08727$ radians

$\cos(10°) \approx 0.9848$

Distance = 6400 × 0.08727 × 0.9848 ≈ 550.336

Answer: 550 km

(a)(ii) Time = 550 / 50 = 11 hours

Answer: 11 hours

(b)(i) Surface area = $2(PQ \times QR + PQ \times RY + QR \times RY)$

$= 2(12 \times 8 + 12 \times 4 + 8 \times 4)$

$= 2(96 + 48 + 32) = 2 \times 176 = 352$

Answer: 352 cm^2

(b)(ii) Planes PTZW and QRZW intersect along WZ

Angle = angle between normals

Normal PTZW: (0, 4, -8)

Normal QRZW: (0, 4, 0)

$\cos \theta = (0 \times 0 + 4 \times 4 + (-8) \times 0) / (\sqrt{(16 + 64)} \times \sqrt{16})$

$= 16 / (\sqrt{80} \times 4) = 16 / (8\sqrt{5} \times 4) = 1 / \sqrt{5}$

$\theta \approx \cos^{-1}(1/\sqrt{5}) \approx 63.43°$

Answer: 63.4°

(c) Slant height = 20 cm, radius = 12 cm

Height = $\sqrt{(20^2 - 12^2)} = \sqrt{(400 - 144)} = \sqrt{256} = 16$ cm

Volume = $(1/3) \times \pi \times r^2 \times h$

$= (1/3) \times 3.14 \times 12^2 \times 16$

$= (1/3) \times 3.14 \times 144 \times 16 = 2411.52$

Answer: 2411.52 cm^3

13. (a) The inverse of a matrix A is (4 3; 5 2). Find the matrix A.

(b) Amani and Asha bought Coca-cola and Pepsi drinks for a farewell party. Amani spent Tshs. 9950 to buy 12 bottles of Coca-cola and 5 bottles of Pepsi drinks. Asha spent Tshs. 8150 to buy 9 bottles of Coca-cola and 5 bottles of Pepsi drinks. Formulate a system of linear equations and hence apply the matrix method to find the price of one bottle of each type of the drinks.

(c) Point A(4,2) is reflected in the line $y + x = 0$ followed by an anticlockwise rotation through 90° about the origin. Find the final image of point A.

(a) $A^{-1} = (4\ 3;\ 5\ 2)$

$\det(A^{-1}) = 4 \times 2 - 3 \times 5 = 8 - 15 = -7$

A = (1/(-7))(2 -3; -5 4) = (-2/7 3/7; 5/7 -4/7)

Answer: (-2/7 3/7; 5/7 -4/7)

(b) Let x = Coca-cola, y = Pepsi

$12x + 5y = 9950$ (1)

$9x + 5y = 8150$ (2)

(1) - (2): $3x = 1800$

$x = 600$

$5y = 9950 - 12 \times 600 = 2750$

$y = 550$

Matrix: (12 5; 9 5)(x; y) = (9950; 8150)

A = (12 5; 9 5), $\det(A) = 12 \times 5 - 5 \times 9 = 15$

Inverse = (1/15)(5 -5; -9 12)

(x; y) = (1/15)(5 -5; -9 12)(9950; 8150)

= (1/15)(600 × 5; 600 × -9 + 550 × 12) = (600; 550)

Answer: Coca-cola: 600 Tshs, Pepsi: 550 Tshs

(c) Reflection in $y + x = 0$: $(x, y) \to (-y, -x)$

$A(4,2) \to (-2, -4)$

Rotation 90° anticlockwise: $(x, y) \to (-y, x)$

$(-2, -4) \to (4, -2)$

Answer: (4, -2)

14. (a) Suppose a function f is defined by $f(x) = (x + 2)^2$, find the domain and range of the inverse of the function f.

(b) A businessman plans to buy at most 210 sacks of Irish and sweet potatoes. Irish potatoes cost shs. 30,000 per sack and sweet potatoes cost shs. 5,000 per sack. He can spend up to shs. 2,500,000 for his business. The profit on a single sack of Irish potatoes is shs. 12,000 and for sweet potatoes is shs. 10,000.

How many sacks of each type of potatoes the businessman will buy in order to realize the maximum profit?

(a) $f(x) = (x + 2)^2$

Domain: $\mathbb{R}$

Range: $y \geq 0$

Inverse: $y = (x + 2)^2$

$x = \pm\sqrt{y} - 2$

Since f is not one-to-one, restrict domain $x \geq -2$

$f^{-1}(x) = \sqrt{x} - 2$

Domain of f^{-1}: $x \geq 0$

Range of f^{-1}: $y \geq -2$

Answer: Domain: $[0, \infty)$, Range: $[-2, \infty)$

(b) Let x = Irish, y = sweet

Maximize: $P = 12000x + 10000y$

Constraints:

$x + y \leq 210$

$30000x + 5000y \leq 2{,}500{,}000 \rightarrow 6x + y \leq 500$

$x \geq 0, y \geq 0$

Vertices:

(0,0): P = 0

(0,210): $P = 10000 \times 210 = 2{,}100{,}000$

(83,2): $6x + y = 500$, $x + y = 210 \rightarrow x = 83$, $y = 127$

$P = 12000 \times 83 + 10000 \times 127 = 996{,}000 + 1{,}270{,}000 = 2{,}266{,}000$

(500/6,0): $P = 12000 \times (500/6) = 1{,}000{,}000$

Maximum at (83,127)

Answer: 83 Irish, 127 sweet