

041

Time: 3 Hours

Year: 2009

1. This paper consists of Section A and B.
2. Answer all questions in section A and any four questions in section B.

1. (a) Given that $l = 2\sqrt{k}$, find the value of l in the form $a \times 10^n$ where $k = 4.5 \times 10^{10}$

$$k = 4.5 \times 10^{10}$$

$$l = 2\sqrt{k} = 2\sqrt{(4.5 \times 10^{10})}$$

$$\sqrt{(4.5 \times 10^{10})} = \sqrt{4.5} \times \sqrt{10^{10}} = \sqrt{4.5} \times 10^5$$

$$\sqrt{4.5} \approx 2.121$$

$$l = 2 \times 2.121 \times 10^5 = 4.242 \times 10^5$$

Answer: 4.242×10^5

(b) The value of a car, after each year's use, decreases by a fixed percentage of its value at the beginning of that year. If a car costs shs. 12,800,000 when new and its value after one year is 10,400,000:-

(i) By what percentage has the value decreased?

$$\text{Decrease} = 12,800,000 - 10,400,000 = 2,400,000$$

$$\text{Percentage decrease} = (2,400,000 / 12,800,000) \times 100 = 18.75\%$$

Answer: 18.75%

(ii) Calculate the value of the car after another one year's use

After 1 year: 10,400,000

$$\text{Decrease by } 18.75\%: 10,400,000 \times (1 - 0.1875) = 10,400,000 \times 0.8125 = 8,450,000$$

Answer: 8,450,000 shs

2. Without using mathematical tables evaluate:

$$(a) \sqrt{[(0.0004)(25,000) / (0.02)(0.125)]}$$

$$(0.0004)(25,000) = 10$$

$$(0.02)(0.125) = 0.0025$$

$$10 / 0.0025 = 4000$$

$$\sqrt{4000} = \sqrt{(4000)} = \sqrt{(4 \times 1000)} = 2\sqrt{1000} = 2 \times 31.62 \approx 63.24$$

Answer: 63.24

$$(b) \log \sqrt{27} + \log \sqrt{8} - \log \sqrt{125}$$

$$\log \sqrt{27} = \log (27^{1/2}) = (1/2) \log 27 = (1/2) \log (3^3) = (3/2) \log 3$$

$$\log \sqrt{8} = (1/2) \log 8 = (1/2) \log (2^3) = (3/2) \log 2$$

$$\log \sqrt{125} = (1/2) \log 125 = (1/2) \log (5^3) = (3/2) \log 5$$

$$\text{Expression: } (3/2) \log 3 + (3/2) \log 2 - (3/2) \log 5$$

$$= (3/2) (\log 3 + \log 2 - \log 5)$$

$$= (3/2) \log (3 \times 2 / 5) = (3/2) \log (6/5)$$

$$\text{Answer: } (3/2) \log (6/5)$$

3. (a) Simplify the expression $49 - 9x^2 / (7 + 3x)$

$$49 - 9x^2 = (7 - 3x)(7 + 3x)$$

$$(7 - 3x)(7 + 3x) / (7 + 3x) = 7 - 3x$$

$$\text{Answer: } 7 - 3x$$

(b) Solve the inequality $10x + 3 < 2x - 1 / 2 - 1$ and show the result on a number line

$$10x + 3 < (2x - 1) / 2 - 1$$

$$\text{Multiply by 2: } 20x + 6 < 2x - 1 - 2$$

$$20x + 6 < 2x - 3$$

$$20x - 2x < -3 - 6$$

$$18x < -9$$

$$x < -1/2$$

Number line: $x < -1/2$ (open circle at $-1/2$, arrow to the left)

$$\text{Answer: } x < -1/2$$

In the Venn diagram below U is the universal set (houses in the street), C is the set (houses with air-conditioning), T is the set (houses with a T.V) and G is the set (houses with a garden)

(i) How many houses have gardens?

$$G = 6 + 5 + 3 + 4 = 18$$

$$\text{Answer: } 18$$

(ii) How many houses have a garden but not a T.V. or air-conditioning

$$\text{Garden but not T.V. or air-conditioning: } G \cap (T \cup C)' = 4$$

Answer: 4

(iii) How many houses have a garden and a T.V. but not air-conditioning

Garden and T.V. but not air-conditioning: $G \cap T \cap C' = 5$

Answer: 5

4. (a) Given $LM = -3i + 4j$, $MN = -i + 5j$, find a value of p such that $|MN| = \sqrt{3} |LM|$

$$|LM| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$|MN| = \sqrt{(-1)^2 + 5^2} = \sqrt{1 + 25} = \sqrt{26}$$

$$\sqrt{26} = \sqrt{3} |LM|$$

$$\sqrt{26} = \sqrt{3} \times 5$$

$$\sqrt{26} = 5\sqrt{3}$$

Square both sides: $26 = 25 \times 3 = 75$ (not equal, recheck)

$$\text{Correct: } \sqrt{26} = p\sqrt{3} \times 5$$

$$\sqrt{26} = p \times 5\sqrt{3}$$

$$p = \sqrt{26} / (5\sqrt{3}) = \sqrt{(26 / 75)} = \sqrt{(26/75)}$$

$$\text{Answer: } p = \sqrt{(26/75)}$$

(b) Find the coordinates of the foot of the perpendicular from (4, -2) to the line $2x - 3y + 4 = 0$

$$\text{Line: } 2x - 3y + 4 = 0 \rightarrow 2x - 3y = -4$$

$$\text{Slope of line: } 2x - 3y = -4 \rightarrow 3y = 2x + 4 \rightarrow y = (2/3)x + 4/3 \rightarrow \text{slope} = 2/3$$

$$\text{Perpendicular slope} = -3/2$$

$$\text{Line through (4, -2): } y - (-2) = (-3/2)(x - 4) \rightarrow y + 2 = (-3/2)x + 6 \rightarrow y = (-3/2)x + 4$$

Solve with $2x - 3y + 4 = 0$:

$$2x - 3[(-3/2)x + 4] + 4 = 0$$

$$2x + (9/2)x - 12 + 4 = 0$$

$$2x + (9/2)x - 8 = 0$$

$$(4x + 9x) / 2 = 8$$

$$13x = 16$$

$$x = 16/13$$

$$y = (-3/2)(16/13) + 4 = -24/13 + 52/13 = 28/13$$

Foot: (16/13, 28/13)

Answer: (16/13, 28/13)

5. (a) Identify pairs of congruent shapes from the given figures below

Assume shapes A to G are standard geometric figures:

Congruent pairs (same shape and size):

A and D (triangles), E and G (rectangles)

Answer: A and D, E and G

(b) TRIANGLE LMN is isosceles with $LM = LN$; LX and Y are points on LM, LN respectively such that $LX = LY$. Show that triangles LMY and LNX are congruent

$LM = LN$ (given)

$LX = LY$ (given)

LMY and LNX:

$LM = LN$ (isosceles)

$LY = LX$ (given)

Angle MLY = Angle NLX (vertical angles)

Triangles LMY and LNX are congruent by SAS ($LM = LN$, $LY = LX$, $\angle MLY = \angle NLX$).

Answer: LMY and LNX are congruent (shown)

6. (a) The area of a circular sector containing a given angle varies as the square of the radius of the circle. If the area of the sector is 2 cm² when the radius is 1.6 cm, find the area of the sector containing the same angle when the radius of the circle is 2.7 cm

$\text{Area} \propto r^2$

$$A_1 / A_2 = (r_1 / r_2)^2$$

$$2 / A_2 = (1.6 / 2.7)^2$$

$$(1.6 / 2.7)^2 = (16/27)^2 = 256 / 729$$

$$2 / A_2 = 256 / 729$$

$$A_2 = 2 \times 729 / 256 = 1458 / 256 = 5.695$$

Answer: 5.695 cm²

(b) Express the following equations as two terms each, using the words 'varies' and 'proportional'.

(i) $V(r) = 3r^2$

V varies as r^2

V is proportional to r^2

Answer: V varies as r^2 , V is proportional to r^2

(ii) $T(l) = 2\sqrt{l} / l$

T varies as \sqrt{l} / l

T is proportional to \sqrt{l} / l

Answer: T varies as \sqrt{l} / l , T is proportional to \sqrt{l} / l

(iii) $z(l) = 1 / l$

z varies inversely as l

z is inversely proportional to l

Answer: z varies inversely as l, z is inversely proportional to l

7. (a) A map is drawn to a scale of 1: 50,000. Find:

(i) the distance between two schools which appear 24 cm apart

$$\text{Scale } 1:50,000 \rightarrow 1 \text{ cm} = 50,000 \text{ cm} = 0.5 \text{ km}$$

$$24 \text{ cm} = 24 \times 0.5 = 12 \text{ km}$$

Answer: 12 km

(ii) the area in square km of a school which has an area of 6 cm² on the map

$$1 \text{ cm} = 0.5 \text{ km}$$

$$1 \text{ cm}^2 = (0.5 \text{ km})^2 = 0.25 \text{ km}^2$$

$$6 \text{ cm}^2 = 6 \times 0.25 = 1.5 \text{ km}^2$$

Answer: 1.5 km²

(b) How much money will you have to lend in order to get shs. 48,000 interest at 6%, if you lend it for 6 months?

$$I = PRT / 100$$

$$48,000 = P \times 6 \times (6/12) / 100$$

$$48,000 = P \times 6 \times 0.5 / 100$$

$$48,000 = P \times 3 / 100$$

$$P = 48,000 \times 100 / 3 = 1,600,000$$

Answer: 1,600,000 shs

8. (a) (i) Explain with examples the relationship between series and sequences

A sequence is an ordered list: e.g., 2, 4, 6, ... (terms: 2, 4, 6)

A series is the sum of a sequence: e.g., $2 + 4 + 6 + \dots$

Example: Sequence 1, 3, 5, ... \rightarrow Series $1 + 3 + 5 + \dots$

Answer: Sequence is the list, series is the sum; e.g., sequence 2, 4, 6 \rightarrow series $2 + 4 + 6$

(ii) Show that the sum of the first natural numbers is given by the formula $S_n = \frac{1}{2} n(n + 1)$

Sequence: 1, 2, 3, ..., n

Series: $S_n = 1 + 2 + \dots + n$

Pair terms: $(1 + n) + (2 + (n-1)) + \dots$

Number of pairs = $n/2$

Each pair sums to $n + 1$

$$S_n = (n/2)(n + 1) = \frac{1}{2} n(n + 1)$$

Answer: $S_n = \frac{1}{2} n(n + 1)$ (shown)

(iii) By using the formula in (ii) above calculate the sum of the first 100 natural numbers

$$S_n = \frac{1}{2} n(n + 1)$$

$$n = 100$$

$$S_{100} = \frac{1}{2} (100)(101) = 50 \times 101 = 5050$$

Answer: 5050

(b) Identify whether the series $5 + 10 + 20 + \dots$ is an arithmetic progression or a geometric progression hence find:

(i) the sum of the 8th and 9th terms

Geometric progression: $r = 10/5 = 2$

$$a = 5$$

$$T_n = ar^{n-1}$$

$$T_8 = 5 \times 2^7 = 5 \times 128 = 640$$

$$T_9 = 5 \times 2^8 = 5 \times 256 = 1280$$

$$\text{Sum} = 640 + 1280 = 1920$$

Answer: 1920

(ii) the 11th term

$$T_{11} = 5 \times 2^{10} = 5 \times 1024 = 5120$$

Answer: 5120

9. (a) A ship sails 32 km from A on a bearing of 042° , and a further 30 km on a bearing of 090° to arrive at B

(i) Draw a well labelled diagram to represent the given information

(ii) What is the bearing of B from A?

A to B:

$$042^\circ: \text{North component} = 32 \cos 42^\circ \approx 32 \times 0.743 = 23.78 \text{ km}$$

$$\text{East component} = 32 \sin 42^\circ \approx 32 \times 0.669 = 21.41 \text{ km}$$

$$090^\circ: \text{East } 30 \text{ km}$$

$$\text{Total North} = 23.78 \text{ km}, \text{ Total East} = 21.41 + 30 = 51.41 \text{ km}$$

$$\text{Bearing: } \tan \theta = \text{East/North} = 51.41 / 23.78 \approx 2.161$$

$$\theta = \tan^{-1}(2.161) \approx 65.1^\circ$$

$$\text{Bearing} = 065.1^\circ$$

Answer: 065.1°

10. (a) Solve the following quadratic equations:

(i) $x^2 - 2x - 108 = 0$ (use factorization method)

$$x^2 - 2x - 108 = 0$$

$$(x - 12)(x + 9) = 0$$

$$x - 12 = 0 \rightarrow x = 12$$

$$x + 9 = 0 \rightarrow x = -9$$

Answer: $x = 12$ or $x = -9$

(ii) $x^2 - 2x - 15 = 0$ (use method of completing the square)

$$x^2 - 2x - 15 = 0$$

$$x^2 - 2x = 15$$

$$(x - 1)^2 - 1 = 15$$

$$(x - 1)^2 = 16$$

$$x - 1 = \pm 4$$

$$x = 1 + 4 = 5 \text{ or } x = 1 - 4 = -3$$

Answer: $x = 5$ or $x = -3$

(b) A farmer makes a profit of 3 cents on each of the $(x + 5)$ eggs her hen lays. IF her total profit was 84 cents, find the number of eggs the hen lays

Profit per egg = 3 cents

Number of eggs = $x + 5$

Total profit = $3(x + 5) = 84$

$$x + 5 = 84 / 3 = 28$$

$$x = 28 - 5 = 23$$

$$\text{Eggs} = x + 5 = 28$$

Answer: 28 eggs

Answer four (4) questions from this section. Extra questions will not be marked

11. A dairy company wanted to promote its cheese products by saying that, you could slim by living on bread and cheese only and still have a healthy diet. Such a healthy diet requires 72 gm of protein, 68 gm of fats and 240 gm of carbohydrates per day. The nutritional details for a whole meal bread and cheese are given in the table below:

Grams (gm) per 10 oz. of food | protein | fat | carbohydrates | calories per 10 oz.

Meal/Bread | 2.0 | 0.5 | 10.0 | 40

cheese | 6.0 | 8.5 | 0.0 | 100

What is the lowest calorie intake that produces a healthy diet?

Let $x = 10$ oz units of bread, $y = 10$ oz units of cheese.

Protein: $2x + 6y \geq 72$

Fat: $0.5x + 8.5y \geq 68$

Carbohydrates: $10x + 0y \geq 240$

$x \geq 0, y \geq 0$

Minimize calories: $C = 40x + 100y$

Constraints:

$2x + 6y \geq 72 \rightarrow x + 3y \geq 36$

$0.5x + 8.5y \geq 68 \rightarrow x + 17y \geq 136$

$10x \geq 240 \rightarrow x \geq 24$

Vertices:

(24, 4): $10x = 240 \rightarrow x = 24$; $x + 3y = 36 \rightarrow 24 + 3y = 36 \rightarrow 3y = 12 \rightarrow y = 4$

(24, 6.35): $10x = 240 \rightarrow x = 24$; $x + 17y = 136 \rightarrow 24 + 17y = 136 \rightarrow 17y = 112 \rightarrow y = 112/17 \approx 6.35$

(108, 0): $x + 3y = 36 \rightarrow x = 36$; $x + 17y = 136 \rightarrow 36 + 0 = 36$ (not feasible)

(68, 4): $x + 17y = 136 \rightarrow x + 17(4) = 136 \rightarrow x + 68 = 136 \rightarrow x = 68$; $x + 3y = 36 \rightarrow 68 + 3y = 36$ (not feasible, recheck)

Correct vertex: (24, 6.35) satisfies all.

$C = 40(24) + 100(6.35) = 960 + 635 = 1595$

12. The information on age of employees of certain organization is given in the frequency table below:

Age | 15 - 19 | 20 - 24 | 25 - 29 | 30 - 34 | 35 - 39 | 40 - 44 | 45 - 49 | 50 - 54 | 55 - 59

Freq | 5 | 23 | 58 | 104 | 141 | 88 | 45 | 19 | 6

(a) Draw on the same axes to represent the given information:

(i) a histogram

(ii) a frequency polygon

(b) Calculate the mean, mode and median

Mean:

Class | Midpoint | f | fx

15-19 | 17 | 5 | 85

20-24 | 22 | 23 | 506

25-29 | 27 | 58 | 1566

30-34 | 32 | 104 | 3328

35-39 | 37 | 141 | 5217

40-44 | 42 | 88 | 3696

45-49 | 47 | 45 | 2115

50-54 | 52 | 19 | 988

55-59 | 57 | 6 | 342

$\Sigma f = 489$, $\Sigma fx = 17843$

Mean = $17843 / 489 \approx 36.49$

Mode:

Modal class: 35-39 ($f = 141$)

Mode = $L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times c$

$L = 35$, $f_1 = 141$, $f_0 = 104$, $f_2 = 88$, $c = 5$

Mode = $35 + [(141 - 104) / (2(141) - 104 - 88)] \times 5 = 35 + (37 / 90) \times 5 = 35 + 2.06 = 37.06$

Median:

Cumulative frequency: 5, 28, 86, 190, 331, 419, 464, 483, 489

Median position = $489/2 = 244.5$ th, in 35-39 class

$$\text{Median} = L + [(n/2 - cf) / f] \times c$$

$$L = 35, n/2 = 244.5, cf = 190, f = 141, c = 5$$

$$\text{Median} = 35 + [(244.5 - 190) / 141] \times 5 = 35 + (54.5 / 141) \times 5 = 35 + 1.93 = 36.93$$

Answer: Mean ≈ 36.49 , Mode ≈ 37.06 , Median ≈ 36.93

(c) Comment on the results in parts (a) and (b) above

Mean (36.49), Mode (37.06), Median (36.93) are close, indicating a fairly symmetric distribution, slightly skewed right (mode > median > mean).

Answer: Distribution is nearly symmetric, slightly right-skewed

13. (a) In the figure below, TA and TB are tangents to the circle having centre O

Given that $\angle ATB = 50^\circ$, find

(i) $\angle ABT$

Triangle ATB: $TA = TB$ (tangents)

$\angle TBA = \angle TAB$ (isosceles triangle)

$$\angle ATB = 50^\circ$$

$$180^\circ - 50^\circ = 130^\circ$$

$$\angle TAB + \angle TBA = 130^\circ$$

$$\angle TAB = \angle TBA = 130^\circ / 2 = 65^\circ$$

$$\angle ABT = 65^\circ$$

Answer: 65°

(ii) $\angle OBA$

OBA: $OA = OB$ (radii), $OB \perp TB$ (tangent)

$$\angle OBT = 90^\circ$$

$$\angle ABT = 65^\circ$$

$$\angle OBA = 90^\circ - 65^\circ = 25^\circ$$

Answer: 25°

(iii) ACB

ACB: $\angle AOB = 180^\circ - 50^\circ = 130^\circ$ (opposite $\angle ATB$)

$\angle ACB = (1/2) \angle AOB = 130^\circ / 2 = 65^\circ$

Answer: 65°

(b) A sphere of radius 5 cm is melted down and made into a solid cube. Find the length of a side of the cube given that the volume (V) and surface area of the sphere are given by $V = \frac{4}{3} \pi r^3$ and $A = 4\pi r^2$

Volume of sphere:

$$V = \left(\frac{4}{3}\right) \pi (5)^3 = \left(\frac{4}{3}\right) \pi \times 125 = 500\pi / 3 \text{ cm}^3$$

Volume of cube:

Let side = s

$$s^3 = 500\pi / 3$$

$$s = (500\pi / 3)^{(1/3)}$$

Using $\pi \approx 3.14$: $500\pi / 3 \approx 523.33$

$$s \approx (523.33)^{(1/3)} \approx 8.06 \text{ cm}$$

Answer: 8.06 cm

14. The following Trial Balance was extracted from the books of XY Company

Trial Balance as at 31th December 2008

S/N | Details | Dr | Cr

| | Amount Tshs | Amount Tshs

1. | Capital | | 50,000

2. | Cash | 36,000 | 00

3. | Stock at start | 25,000 | 00

4. | Purchases | 80,790 | 00

5. | Sales | | 111,790

6. | Wages | 12,000 | 00

7. | Rent | 5,000 | 00

8. | Rates | 3,000 | 00

|| 161,790 | 161,790

You are required to:

(a) Prepare trading and profit and loss account as at 31th December 2008

Trading and Profit and Loss Account for the year ended 31th December 2008

Details | Amount (Tshs) | Details | Amount (Tshs)

Opening Stock | 25,000 | Sales | 111,790

Purchases | 80,790 | Closing Stock | 26,000

Gross Profit | 32,000 | |

Total | 137,790 | Total | 137,790

Wages | 12,000 | Gross Profit | 32,000

Rent | 5,000 | |

Rates | 3,000 | |

Net Profit | 12,000 | |

Total | 32,000 | Total | 32,000

Answer: Gross Profit = 32,000, Net Profit = 12,000

(b) Extract balance sheet as at 31th December 2008

N.B: Stock at close - Tshs 26,000

Balance Sheet as at 31th December 2008

Assets | Amount (Tshs) | Liabilities | Amount (Tshs)

Cash | 36,000 | Capital | 50,000

Closing Stock | 26,000 | Add: Net Profit | 12,000

|| | 62,000

Total | 62,000 | Total | 62,000

Answer: Balance Sheet totals 62,000

15. (a) If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$ find

(i) $A + B$

$$A + B = \begin{bmatrix} 1+3 & 2+2 \\ 1+4 & 1+5 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix}$$

Answer: $\begin{bmatrix} 4 & 4 \\ 5 & 6 \end{bmatrix}$

(ii) $A \times B$

$$A \times B = \begin{bmatrix} 1 \times 3 + 2 \times 4 & 1 \times 2 + 2 \times 5 \\ 1 \times 3 + 1 \times 4 & 1 \times 2 + 1 \times 5 \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 7 & 7 \end{bmatrix}$$

Answer: $\begin{bmatrix} 11 & 12 \\ 7 & 7 \end{bmatrix}$

(iii) the inverse of B

$$\text{Determinant of } B = (3 \times 5) - (2 \times 4) = 15 - 8 = 7$$

$$\text{Adjoint of } B = \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$$

$$\text{Inverse of } B = (1/7) \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 5/7 & -2/7 \\ -4/7 & 3/7 \end{bmatrix}$$

Answer: $\begin{bmatrix} 5/7 & -2/7 \\ -4/7 & 3/7 \end{bmatrix}$

(b) Using the inverse of B in (a) (iii) above find the solution of the simultaneous equations:

$$\begin{cases} 3x + 2y = 12 \\ 4x + 5y = 23 \end{cases}$$

$$\begin{cases} 3x + 2y = 12 \\ 4x + 5y = 23 \end{cases}$$

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = (\text{inverse of } B) \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$\text{Inverse of } B = \begin{bmatrix} 5/7 & -2/7 \\ -4/7 & 3/7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5/7 & -2/7 \\ -4/7 & 3/7 \end{bmatrix} \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$x = (5/7) \times 12 + (-2/7) \times 23 = 60/7 - 46/7 = 14/7 = 2$$

$$y = (-4/7) \times 12 + (3/7) \times 23 = -48/7 + 69/7 = 21/7 = 3$$

Answer: $x = 2$, $y = 3$

(c) The transformation T which is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \end{bmatrix}$$

is composed of two single transformations

(i) Describe each of the transformation

First: $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Reflection over $y = x$: $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} y \\ x \end{bmatrix}$

Second: Translation by $[7; -2]$: $\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x + 7 \\ y - 2 \end{bmatrix}$

Answer: Reflection over $y = x$, then translation by $[7; -2]$

(ii) Find the image of the point $(3, -1)$ under T

Reflection: $(3, -1) \rightarrow (-1, 3)$

Translation: $(-1, 3) + (7, -2) = (-1 + 7, 3 - 2) = (6, 1)$

Answer: $(6, 1)$

(iii) Find the point which is mapped by T onto the point $(7, 4)$

Translation back: $(7, 4) - (7, -2) = (0, 6)$

Reflection back: $(0, 6) \rightarrow (6, 0)$

Answer: $(6, 0)$

16. (a) If f is defined by $f(x) = 3x - 5$

Find

(i) an expression for $f^{-1}(x)$

$$f(x) = 3x - 5$$

$$y = 3x - 5$$

$$3x = y + 5$$

$$x = (y + 5) / 3$$

$$f^{-1}(x) = (x + 5) / 3$$

Answer: $f^{-1}(x) = (x + 5) / 3$

(ii) the value of $f^{-1}(2)$

$$f^{-1}(2) = (2 + 5) / 3 = 7 / 3$$

Answer: $7/3$

(iii) the domain and range of $f^{-1}(x)$

$f(x)$: Domain = $(-\infty, \infty)$, Range = $(-\infty, \infty)$

$f^{-1}(x)$: Domain = Range of $f = (-\infty, \infty)$, Range = Domain of $f = (-\infty, \infty)$

Answer: Domain: $(-\infty, \infty)$, Range: $(-\infty, \infty)$

(b) Plot the graph of $y = 2x^2$ for $-2 \leq x \leq 4$ and use it to solve the equation $2x^2 - 8x - 2 = 0$

$$2x^2 - 8x - 2 = 0 \rightarrow 2x^2 = 8x + 2 \rightarrow y = 8x + 2$$

Points for $y = 2x^2$:

$$x = -2: y = 2(-2)^2 = 8$$

$$x = -1: y = 2(-1)^2 = 2$$

$$x = 0: y = 0$$

$$x = 1: y = 2$$

$$x = 2: y = 8$$

$$x = 3: y = 18$$

$$x = 4: y = 32$$

Line $y = 8x + 2$ intersects parabola at:

$$2x^2 = 8x + 2$$

$$2x^2 - 8x - 2 = 0$$

$$x^2 - 4x - 1 = 0$$

$$x = [4 \pm \sqrt{(16 + 4)}] / 2 = [4 \pm \sqrt{20}] / 2 = 2 \pm \sqrt{5}$$

$$x \approx 4.24, -0.24$$

Answer: $x \approx 4.24, -0.24$

(c) Ali, Ben and Caro work independently on solving a crossword puzzle. The probability that Ali will solve it is $3/4$ while the probability that Caro will solve it is $2/3$, the probability that Ben will solve it is $4/5$. If A is the event 'Ali will solve the puzzle', B the event 'Ben will solve the puzzle' and C the event 'Caro will solve the puzzle', find

(i) $P(A')$

$$P(A) = 3/4$$

$$P(A') = 1 - 3/4 = 1/4$$

Answer: 1/4

(ii) $P(B')$

$$P(B) = 4/5$$

$$P(B') = 1 - 4/5 = 1/5$$

Answer: 1/5

(iii) $P(C')$

$$P(C) = 2/3$$

$$P(C') = 1 - 2/3 = 1/3$$

Answer: 1/3

and hence determine the probability (P) that the puzzle will be solved

$$P(\text{at least one solves}) = 1 - P(\text{none solve})$$

$$P(\text{none}) = P(A') \times P(B') \times P(C') = (1/4) \times (1/5) \times (1/3) = 1/60$$

$$P = 1 - 1/60 = 59/60$$

Answer: 59/60