THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(Private Candidates Only)

Time: 3 Hours **ANSWERS** **Year: 2015**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Given that $\sqrt{2} = 1.4142$, evaluate to 4 significant figures:

(i) $\sqrt{12800}$

(ii) $\sqrt{0.0512}$

Answer:

(i) $\sqrt{12800} = \sqrt{(128 \times 100)}$

$= \sqrt{128} \times \sqrt{100}$

$= \sqrt{(64 \times 2)} \times 10$

$= \sqrt{64} \times \sqrt{2} \times 10$

$= 8 \times 1.4142 \times 10$

$= 11.3136 \times 10$

$= 113.136$

To 4 significant figures = 113.1

(ii) $\sqrt{0.0512} = \sqrt{(512 \div 10000)}$

$= \sqrt{512} \div 100$

$= \sqrt{(256 \times 2)} \div 100$

$= \sqrt{256} \times \sqrt{2} \div 100$

$= 16 \times 1.4142 \div 100$

$= 22.6272 \div 100$

$= 0.226272$

To 4 significant figures = 0.2263

Final Answer: (i) 113.1, (ii) 0.2263

(b) Evaluate $324\times10^{-7} \div 15\times10^{-4} + 75\times10^{-3}$ and write the answer in standard form.

Answer:

$324\times10^{-7} \div 15\times10^{-4} + 75\times10^{-3}$

First term: $324\times10^{-7} \div 15\times10^{-4}$

$= (324 \div 15) \times 10^{-7-(-4)}$

$= 21.6 \times 10^{-7+4}$

$= 21.6 \times 10^{-3}$

= 0.0216

Second term: $75\times10^{-3} = 0.075$

0.0216 + 0.075 = 0.0966

Standard form: $0.0966 = 9.66\times10^{-2}$

Final Answer: 9.66×10^{-2}

2. (a) Make x the subject of the equation (1/x) + (1/y) = (1/5)

Answer:

(1/x) + (1/y) = (1/5)

(y + x) ÷ (xy) = 1/5

5(y + x) = xy

5y + 5x = xy

xy - 5x = 5y

x(y - 5) = 5y

x = 5y ÷ (y - 5)

Final Answer: x = 5y ÷ (y - 5)

(b) Find the value of x in the equation: 12 log x = √2 + 8log 27 = 25.

Answer:

12 log x = √2 + 8log 27 = 25

Assume typo, likely: 12 log x = √2 + 8 log 27 - 25

√2 = 1.4142 (given previously)

log 27 = log (3^3) = 3 log 3

log 3 ≈ 0.4771

3 log 3 ≈ 3 × 0.4771 = 1.4313

$8 \log 27 = 8 \times 1.4313 = 11.4504$

Right side: $1.4142 + 11.4504 - 25 = 12.8646 - 25 = -12.1354$

$12 \log x = -12.1354$

$\log x = -12.1354 \div 12 \approx -1.0113$

$x = 10^{-1.0113}$

$10^{-1} = 0.1$

$10^{-0.0113} \approx 0.974$ (approximate)

$x \approx 0.1 \times 0.974 \approx 0.0974$

Final Answer: $x \approx 0.0974$

3. (a) In three years to come Jeremiah will be as old as each of their ages is 23 years, find the age of each.

Answer:

Assume typo: "In three years to come, the sum of their ages will be 23 years."

Let Jeremiah's age be J, and another person's age be A.

In 3 years:

$J + 3 + A + 3 = 23$

$J + A + 6 = 23$

$J + A = 17$

Also: $J + 3 = A + 3$ (Jeremiah will be as old as A)

$J = A$

$J + J = 17$

$2J = 17$

$J = 8.5$

$A = 8.5$

Current ages: $J = 8.5$ years, $A = 8.5$ years

Final Answer: Both are 8.5 years old

(b) Jamungo secondary school has a total of 500 students of which 350 can speak English and 300 can speak French. By using a venn diagram, find how many can speak:

(i) Both French and English,

(ii) English but not French,

(iii) French but not English.

Answer:

Total students = 500

English (E) = 350

French (F) = 300

(i) Both French and English ($E \cap F$):

$|E \cup F| = |E| + |F| - |E \cap F|$

$500 = 350 + 300 - |E \cap F|$

$|E \cap F| = 650 - 500 = 150$

(ii) English but not French:

$|E - F| = |E| - |E \cap F| = 350 - 150 = 200$

(iii) French but not English:

$|F - E| = |F| - |E \cap F| = 300 - 150 = 150$

Final Answer: (i) 150, (ii) 200, (iii) 150

4. (a) Given the position vectors OA = -2i + 4j, OB = 2i + j and OC = 3i - j

(i) Draw on the same pair of axes these vectors and the vector CB.

(ii) Describe the relationship between the vectors OA and CB.

Answer:

(i) Vectors:

OA = (-2, 4)

OB = (2, 1)

OC = (3, -1)

Vector CB = OB - OC

= (2i + j) - (3i - j)

= (2 - 3)i + (1 - (-1))j

= -i + 2j

vectors are:

OA from (0,0) to (-2,4)

OB from (0,0) to (2,1)

OC from (0,0) to (3,-1)

CB from C(3,-1) to B(2,1), or as -i + 2j from origin.

(ii) Relationship between OA and CB:

OA = -2i + 4j

CB = -i + 2j

CB = (1/2) × OA (same direction, half magnitude)

Final Answer: (ii) CB is half of OA in the same direction

(b) Juma walks 25 m North East from his classroom to the football ground and then he walks 20 m due East to the Library.

(i) Represent this information diagrammatically,

(ii) Find the distance between the classroom and the Library.

Answer:

(i) description:

Classroom at origin (0,0).

North East = 45° angle.

25 m NE: $x = 25 \cos 45° = 25 \times (\sqrt{2}/2) \approx 17.68$ m, $y = 25 \sin 45° \approx 17.68$ m.

Football ground at (17.68, 17.68).

20 m East: $x = 17.68 + 20 = 37.68$, $y = 17.68$.

Library at (37.68, 17.68).

(ii) Distance from classroom (0,0) to Library (37.68, 17.68):

Distance = $\sqrt{(37.68^2 + 17.68^2)}$

$37.68^2 \approx 1419.78$

$17.68^2 \approx 312.58$

1419.78 + 312.58 = 1732.36

$\sqrt{1732.36} \approx 41.62$

Final Answer: (ii) 41.62 m

5. (a) (i) Are congruent triangles also similar? Explain.

(ii) Show whether triangles DCF and EBF in the following figure are congruent and/or similar.

[Figure: Triangle DCE with F on CE, DC = EB, ∠DCF = ∠EBF]

Answer:

(i) Yes, congruent triangles are similar. Congruent triangles have equal angles and sides, so their corresponding sides are proportional (ratio 1:1), satisfying the definition of similarity.

(ii) Triangles DCF and EBF:

Given: DC = EB, ∠DCF = ∠EBF, F on CE.

∠DCF = ∠EBF (given).

∠DFC = ∠BFE (vertical angles).

DC = EB (given).

By ASA (Angle-Side-Angle), ΔDCF ≅ ΔEBF.

Since congruent, they are also similar.

Final Answer: (i) Yes, (ii) Congruent and similar

(b) Find the perimeter and the area of a regular six – sided polygon inscribed in a circle of radius 10 cm.

Answer:

Regular hexagon inscribed in circle, radius r = 10 cm.

Side length of hexagon = radius = 10 cm (property of regular hexagon).

Perimeter = $6 \times 10 = 60$ cm

Area: Split into 6 equilateral triangles.

Area of one triangle = $(\sqrt{3}/4) \times (\text{side})^2$

$= (\sqrt{3}/4) \times 10^2$

$= (\sqrt{3}/4) \times 100$

$\sqrt{3} \approx 1.732$

$(1.732 \times 100) \div 4 \approx 43.3$

Total area = $6 \times 43.3 = 259.8$ cm²

Final Answer: Perimeter = 60 cm, Area = 259.8 cm²

6. (a) The variable y varies directly as x and inversely as square of z.

(i) Write the equation connecting y, x and z.

(ii) If x is increased by 5% and z is decreased by 10% write down the new equation connecting y, x and z hence find the percentage change in y.

Answer:

(i) $y \propto x/z^2$

$y = kx/z^2$

(ii) x increases by 5%: $x' = x \times 1.05$

z decreases by 10%: $z' = z \times 0.9$

New equation: $y' = k(x \times 1.05)/(z \times 0.9)^2$

$= k(1.05x)/(0.81z^2)$

$= (1.05/0.81) \times (kx/z^2)$

$= (1.05/0.81) \times y$

$1.05 \div 0.81 \approx 1.2963$

$y' = 1.2963y$

Percentage change in y:

$(1.2963 - 1) \times 100 \approx 29.63\%$

Final Answer: (i) $y = kx/z^2$, (ii) $y' = (1.05/0.81)kx/z^2$, 29.63% increase

(b) It takes 24 days for 20 people to accomplish a certain task. How long could it take for 30 people to complete the same task?

Answer:

Work rate: 20 people in 24 days.

Total work = $20 \times 24 = 480$ person-days.

30 people: Days = $480 \div 30 = 16$

Final Answer: 16 days

7. (a) The ratio of boys to girls at Chipelo secondary school is 3:7. IF the school has 500 students, find the number of boys at the school.

Answer:

Ratio boys:girls = 3:7

Total parts = $3 + 7 = 10$

Total students = 500

Boys = $(3/10) \times 500 = 150$

Final Answer: 150 boys

(b) Jerry sold his computer for sh. 24,300 and as a result lost 20% of the price he paid for it.

(i) How much did he pay for the computer?

(ii) What was the loss he incurred?

Answer:

(i) Selling price = 80% of cost price (20% loss)

Let cost price = C

$0.8C = 24300$

C = 24300 ÷ 0.8 = 30375

(ii) Loss = 20% of C

= 0.2 × 30375

= 6075

Final Answer: (i) sh. 30375, (ii) sh. 6075

8. (a) After completing Form Four Safina will work for Tazima Company with a starting annual salary of 1,500,000 shillings. If the company offers an annual increment of 50,000 shillings, how much will she get after nine years?

Answer:

Annual salary forms an A.P.

First term a = 1500000

Common difference d = 50000

After 9 years, term n = 9

nth term = a + (n-1)d

9th term = 1500000 + (9-1) × 50000

= 1500000 + 8 × 50000

= 1500000 + 400000

= 1900000

Final Answer: sh. 1900000

(b) Find the sum of all the multiples of 3 between 1 and 201.

Answer:

Multiples of 3: 3, 6, ..., 201

Last term ≤ 201: 201 ÷ 3 = 67

Terms: 3, 6, ..., 201 (A.P.)

Number of terms n = 67

First term $a = 3$, last term $l = 201$

Sum $= n(a + l)/2$

$= 67 \times (3 + 201)/2$

$= 67 \times 204/2$

$= 67 \times 102$

$= 6834$

Final Answer: 6834

9. (a) Use the mathematical tables to find sine, cosine and tangent of 108°.

Answer:

Mathematical tables not provided, approximate:

$108° = 180° - 72°$

$\sin 108° = \sin 72°$

$\cos 108° = -\cos 72°$

$\tan 108° = -\tan 72°$

From standard values (approximate):

$\sin 72° \approx 0.9511$

$\cos 72° \approx 0.3090$

$\tan 72° = \sin 72°/\cos 72° \approx 0.9511/0.3090 \approx 3.077$

$\sin 108° \approx 0.9511$

$\cos 108° \approx -0.3090$

$\tan 108° \approx -3.077$

Final Answer: $\sin 108° \approx 0.9511$, $\cos 108° \approx -0.3090$, $\tan 108° \approx -3.077$

(b) A car travels 180 m along a straight road which is inclined at 10.8° to the horizontal. Calculate the vertical distance through which the car rises.

Answer:

Angle $\theta = 10.8°$

Hypotenuse = 180 m

Vertical distance = $180 \times \sin 10.8°$

$\sin 10.8° \approx \sin 10°$ (0.1736) + adjustment

$\sin 10.8° \approx 0.1874$ (approximate)

Vertical = $180 \times 0.1874 \approx 33.732$

Final Answer: 33.73 m

(c) (i) State Pythagoras' theorem.

(ii) Find the sides of a square which has the diagonals of length 20 cm, correct to four decimal places.

Answer:

(i) Pythagoras' theorem: In a right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.

(ii) Square diagonal = 20 cm

Let side = s

Diagonal = $s\sqrt{2}$

$s\sqrt{2} = 20$

$s = 20/\sqrt{2}$

$\sqrt{2} \approx 1.4142$

$s = 20/1.4142 \approx 14.1421$

Final Answer: (i) $a^2 + b^2 = c^2$, (ii) 14.1421 cm

10. (a) Solve the simultaneous equations: $3x^2 + 12y^2 = 6$ and $2x + 4y = 1$

Answer:

$3x^2 + 12y^2 = 6$ (1)

$2x + 4y = 1$ (2)

Simplify (1): $x^2 + 4y^2 = 2$ (3)

From (2): $2x + 4y = 1$

$x + 2y = 1/2$

$x = 1/2 - 2y$ (4)

Substitute (4) into (3):

$(1/2 - 2y)^2 + 4y^2 = 2$

$(1/4 - 2y + 4y^2) + 4y^2 = 2$

$1/4 - 2y + 4y^2 + 4y^2 = 2$

$8y^2 - 2y + 1/4 = 2$

$8y^2 - 2y + 1/4 - 2 = 0$

$8y^2 - 2y - 7/4 = 0$

$32y^2 - 8y - 7 = 0$

Solve: $y = (8 \pm \sqrt{(64 + 896)})/64$

$= (8 \pm \sqrt{960})/64$

$\sqrt{960} \approx 30.9839$

$y = (8 \pm 30.9839)/64$

$y \approx 0.609$ or $y \approx -0.359$

$y \approx 0.609$: $x = 1/2 - 2(0.609) = -0.718$

$y \approx -0.359$: $x = 1/2 - 2(-0.359) = 1.218$

Final Answer: $(x, y) \approx (-0.718, 0.609)$ or $(1.218, -0.359)$

(b) Two numbers differ by 7. If their product is 60, find the numbers.

Answer:

Let numbers be x and y, $x - y = 7$, $xy = 60$

$x = y + 7$

$(y + 7)y = 60$

$y^2 + 7y - 60 = 0$

$(y + 12)(y - 5) = 0$

y = -12 or y = 5

y = -12: x = -12 + 7 = -5

y = 5: x = 5 + 7 = 12

Final Answer: -5 and -12, or 12 and 5

11. (a) Ally wishes to buy up to 40 notebooks for his stationery. He can buy either type A for sh. 3,000 each or type B for sh. 6,000 each. He has a total of sh. 150,000 to spend and he have at least 10 notebooks of type A and at least 5 notebooks of type B in his stock. Write down all the inequality which represent the given information.

Answer:

Let x = number of type A notebooks, y = number of type B notebooks

$x + y \leq 40$ (total notebooks)

$3000x + 6000y \leq 150000 \rightarrow x + 2y \leq 50$ (total cost)

$x \geq 10$ (minimum type A)

$y \geq 5$ (minimum type B)

$x \geq 0, y \geq 0$ (non-negative)

Final Answer: $x + y \leq 40$, $x + 2y \leq 50$, $x \geq 10$, $y \geq 5$, $x \geq 0$, $y \geq 0$

(b) If he makes a profit of sh. 400 on each notebook of type A and sh. 1,000 on each notebook of type B, how many notebooks of each type he should buy for maximum profit?

Answer:

Profit $P = 400x + 1000y$

Constraints:

$x + y \leq 40$

$x + 2y \leq 50$

$x \geq 10$

$y \geq 5$

Vertices of feasible region:

(10, 5): 400×10 + 1000×5 = 4000 + 5000 = 9000

(10, 20): x + 2y = 50 → 10 + 2y = 50 → 2y = 40 → y = 20

400×10 + 1000×20 = 4000 + 20000 = 24000

(35, 5): x + y = 40 → x + 5 = 40 → x = 35

400×35 + 1000×5 = 14000 + 5000 = 19000

(30, 10): x + y = 40, x + 2y = 50 → 2y - y = 50 - 40 → y = 10, x = 30

400×30 + 1000×10 = 12000 + 10000 = 22000

Max profit at (10, 20): 24000

Final Answer: 10 type A, 20 type B

12. (a) Mode, median and mean are the measures of central tendency of a distribution. Give a description of each term.

Answer:

Mode: The value that appears most frequently in the data.

Median: The middle value when data is ordered.

Mean: The average, sum of all values divided by the number of values.

Final Answer: Mode: most frequent value, Median: middle value, Mean: average

(b) The following frequency distribution table shows the marks of 100 students in an end of term Mathematics examination.

Mark (%)	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
Frequency	11	23	20	17	18	7	4

(i) How many students had less than 71 marks?

(ii) How many students had at least 41 marks?

(iii) Determine the modal and the median classes

(iv) Determine an estimate of the mean of the marks.

(v) Draw a cumulative frequency curve of the marks.

(vi) Estimate the median examination mark from the graph.

Answer:

(i) Less than 71: 11 + 23 + 20 + 17 = 71

(ii) At least 41: 23 + 20 + 17 + 18 + 7 + 4 = 89

(iii) Modal class: 41–50 (highest frequency 23)

Median class: 50th student (cumulative freq: 11, 34, 54), 51–60

(iv) Mean:

Midpoints: 35.5, 45.5, 55.5, 65.5, 75.5, 85.5, 95.5

Sum = $(35.5\times11) + (45.5\times23) + (55.5\times20) + (65.5\times17) + (75.5\times18) + (85.5\times7) + (95.5\times4)$

= 390.5 + 1046.5 + 1110 + 1113.5 + 1359 + 598.5 + 382

= 6000

Mean = $6000 \div 100 = 60$

(v) cumulative frequencies:

≤40: 11

≤50: 11 + 23 = 34

≤60: 34 + 20 = 54

≤70: 54 + 17 = 71

≤80: 71 + 18 = 89

≤90: 89 + 7 = 96

≤100: 96 + 4 = 100

(vi) Median from graph: 50th student in 51–60 class

Median $\approx 50 + (50 - 34) \div 20 \times 10 = 50 + 16 \div 20 \times 10 = 50 + 8 = 58$

Final Answer: (i) 71, (ii) 89, (iii) Modal: 41–50, Median: 51–60, (iv) 60, (v) Cumulative frequencies listed, (vi) 58

13. (a) A bucket UVST was made from the cone URT such that its top radius is 12 cm and its depth (PQ) is 10 cm.

(i) Find the height QR of the cone VRS, using the fact that triangles PRT and QRS are similar.

(ii) Find the volume of the bucket, giving your answer in four significant figures.

Answer:

(i) Triangles PRT and QRS similar:

PR = 12 cm, PQ = 10 cm, QS = 10 cm

PQ/QS = PR/RS

10/10 = 12/RS

RS = 12 cm

In ΔQRS, $QR^2 = QS^2 + RS^2$

$QR^2 = 10^2 + 12^2 = 100 + 144 = 244$

$QR = \sqrt{244} \approx 15.62$ cm

(ii) Volume of cone VRS:

$V = (1/3)\pi r^2 h = (1/3)\pi(12)^2(15.62) = (1/3)\pi\times144\times15.62 \approx 2357.76\ cm^3$

Volume of cone VUT (top radius 12 cm, height 10 cm):

$V = (1/3)\pi(12)^2(10) = (1/3)\pi\times144\times10 \approx 1507.96\ cm^3$

Volume of bucket = 2357.76 - 1507.96 = 849.8 cm^3

To 4 significant figures: 849.8

Final Answer: (i) 15.62 cm, (ii) 849.8 cm^3

(b) In the diagram below PA and PB are tangents to the circle. If O is the centre of the circle, find the values of x and y.

Answer:

PA and PB are tangents, $OA \perp PA$, $OB \perp PB$

$\angle OAP = \angle OBP = 90°$

In ΔOAP:

$OP = 20$ cm, $PA = 16$ cm

$OA = \sqrt{(OP^2 - PA^2)} = \sqrt{(20^2 - 16^2)} = \sqrt{(400 - 256)} = \sqrt{144} = 12$ cm

$\angle OPA = x$ (same in ΔOBP by symmetry)

$\sin x = OA/OP = 12/20 = 0.6$

$x = \sin^{-1}(0.6) \approx 36.87°$

$\angle APB = y$

$\angle APO + \angle BPO = y$ (since $\angle APO = \angle BPO = 90° - x$)

$y = 2(90° - x) = 2(90° - 36.87°) = 2\times53.13° \approx 106.26°$

Final Answer: $x \approx 36.87°$, $y \approx 106.26°$

(c) Find the distance along the circle of latitude between A(32°15', 43°5') and B(28°15', 43°5'). (Use the radius of the earth R = 6400 km).

Answer:

Points on same latitude $43°5' = 43 + 5/60 = 43.0833°$

Longitude difference:

$32°15' = 32 + 15/60 = 32.25°$

$28°15' = 28 + 15/60 = 28.25°$

$\Delta\theta = 32.25° - 28.25° = 4°$

Distance along latitude:

$s = r\theta \cos(\text{latitude})$

θ in radians: $4° \times \pi/180 \approx 0.06981$

$s = 6400 \times 0.06981 \times \cos 43.0833°$

$\cos 43.0833° \approx 0.7298$

$s \approx 6400 \times 0.06981 \times 0.7298 \approx 326.2$ km

Final Answer: 326.2 km

14. (a) What is an account?

Answer:

An account is a record of financial transactions, tracking income, expenses, assets, and liabilities.

Final Answer: A record of financial transactions

(b) State the principle of double entry.

Answer:

Every financial transaction affects at least two accounts, with equal debits and credits.

Final Answer: Every transaction affects two accounts with equal debits and credits

(c) What is a trial balance?

Answer:

A trial balance is a list of all accounts and their balances to check that total debits equal total credits.

Final Answer: A list of accounts to check debits equal credits

(d) On 1st August, 2001 Mr Paulo started business with capital in cash 800,000/=

4 Paid salary for cash 150,000/=

8 Sold goods for cash 200,000/=

12 Paid rent for cash 220,000/=

18 Purchased goods for cash 300,000/=

28 Sold goods for cash 350,000/=

29 Paid insurance for cash 250,000/=

Enter the above transactions in a cash account, balance it and bring down the balance to the next month.

Answer:

Cash Account:

Date | Details | Amount | Date | Details | Amount

Aug 1 | Capital | 800000 | Aug 4 | Salary | 150000

Aug 8 | Sales | 200000 | Aug 12 | Rent | 220000

Aug 28 | Sales | 350000 | Aug 18 | Purchases | 300000

Aug 29 | Insurance | 250000

| | | Aug 31 | Balance c/d | 430000

Total | | 1350000 | Total | | 1350000

Sep 1 | Balance b/d | 430000

Final Answer: Balance on Sep 1: 430000

15. (a) If A and B are square matrices of order 2×2, expand the brackets for (A + B)(A - B).

Answer:

(A + B)(A - B) = A(A - B) + B(A - B)

$= A^2 - AB + BA - B^2$

Final Answer: $A^2 - AB + BA - B^2$

(b) Work out the values of x and y in the following cases:

(i) (5 7)(2 3) = (x y)

(6 8)(x y) = (24 46)

Answer:

(i) (5 7)(2 3) = (5×2 + 7×x 5×3 + 7×y)

(6 8)(x y) = (6×2 + 8×x 6×3 + 8×y)

Given: (10 + 7x 15 + 7y) = (x y)

10 + 7x = x → 10 = x - 7x → 10 = -6x → x = -10/6 = -5/3

15 + 7y = y → 15 = y - 7y → 15 = -6y → y = -15/6 = -5/2

Final Answer: x = -5/3, y = -5/2

(b) (ii) (6 4)(x y) = (24)

(8 10)(y 46) = (46)

Answer:

(6 4)(x y) = (6x + 4y)

(8 10)(y 46) = (8y + 10×46)

Given: 6x + 4y = 24 → 3x + 2y = 12 (1)

8y + 10×46 = 46 → 8y + 460 = 46 → 8y = 46 - 460 → 8y = -414 → y = -414/8 = -51.75 (2)

Substitute y = -51.75 into (1):

3x + 2(-51.75) = 12

3x - 103.5 = 12

3x = 12 + 103.5 = 115.5

x = 115.5/3 = 38.5

Final Answer: x = 38.5, y = -51.75

(c) Find the equation of the line joining points (1, 7) and (2, 9) after a reflection in the line y = x.

Answer:

Slope of line: (9 - 7)/(2 - 1) = 2/1 = 2

Equation: y - 7 = 2(x - 1) → y = 2x + 5

Reflection in y = x: Swap x and y

x = 2y + 5

2y = x - 5

y = (x - 5)/2

Final Answer: y = (x - 5)/2

16. (a) The function f is defined as follows:

f(x) = { -x + 2 if x < -1

{ 2 if -1 < x ≤ 1

{ x if x > 1

(i) Sketch the graph of f(x).

(ii) Use the graph to determine the domain and range of f(x).

Answer:

(i) Diagram description:

x < -1: Line y = -x + 2 (at x = -1, y = 3; x = -2, y = 4)

$-1 < x \leq 1$: Horizontal line $y = 2$

$x > 1$: Line $y = x$ (at $x = 1, y = 1; x = 2, y = 2$)

(ii) Domain: All real numbers $(-\infty, \infty)$

Range: $y \geq 2$ for $x < -1$, $y = 2$ for $-1 < x \leq 1$, $y > 1$ for $x > 1$

Overall range: $y \geq 2$ (from $-x + 2$) $\cup$ $y > 1$ (from x), so $y > 1$

Final Answer: (i) Described graph, (ii) Domain: $(-\infty, \infty)$, Range: $(1, \infty)$

(b) A die and a coin are thrown together. If the die has its six faces marked 0, 1, 1, 1, 6, 6 use a tree diagram to determine the probability that:

(i) a tail and a face marked 1 occurs,

(ii) a head and a face marked 1 or a head and a face marked 6 will occur.

Answer:

Coin: H (1/2), T (1/2)

Die: 0 (1/6), 1 (3/6 = 1/2), 6 (2/6 = 1/3)

(i) Tail and face 1:

P(T) = 1/2, P(1) = 1/2

P(T and 1) = (1/2) × (1/2) = 1/4

(ii) Head and face 1 or Head and face 6:

P(H and 1) = P(H) × P(1) = (1/2) × (1/2) = 1/4

P(H and 6) = P(H) × P(6) = (1/2) × (1/3) = 1/6

P(H and 1 or H and 6) = (1/4) + (1/6) = 3/12 + 2/12 = 5/12

Final Answer: (i) 1/4, (ii) 5/12