THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL

DIPLOMA IN SECONDARY EDUCATION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours. **ANSWER** **Year: 2011**

Instructions

1. This paper consists of sections A, B and C.
2. Answer **all** questions from Section A and **two (2)** questions from each of section B and C.
3. Section A and B carry **30** marks, Section C carry 40 marks.
4. Mathematical tables and non-programmable calculators may be used
4. Cellular phones are **not** allowed inside the examination room.
5. Write your **Examination Number** on every page of your answer booklet

SECTION A (30 Marks)

Answer all questions in this section.

1.(a) If vector v of a magnitude 27 units is parallel to vector a = i + 8j + 4k. Determine the value of vector v.

Since v is parallel to a,

$v = \lambda a$

Magnitude of a

$|a| = \sqrt{(1^2 + 8^2 + 4^2)}$

$|a| = \sqrt{(1 + 64 + 16)}$

$|a| = \sqrt{81}$

$|a| = 9$

Given $|v| = 27$

$|v| = |\lambda||a|$

$27 = |\lambda| \times 9$

$|\lambda| = 3$

Therefore

$v = 3(i + 8j + 4k)$

$v = 3i + 24j + 12k$

(b) Find the gradient of the graph of $3\sqrt{(x^2 + y^2)} = 50xy$ at the point (2, 1).

$3\sqrt{(x^2 + y^2)} = 50xy$

Differentiate implicitly with respect to x

$3 \times 1/(2\sqrt{(x^2 + y^2)}) \times (2x + 2y\ dy/dx) = 50(y + x\ dy/dx)$

Simplify

$3(x + y\ dy/dx)/\sqrt{(x^2 + y^2)} = 50y + 50x\ dy/dx$

Substitute $x = 2, y = 1$

$\sqrt{(2^2 + 1^2)} = \sqrt{5}$

$3(2 + dy/dx)/\sqrt{5} = 50 + 100\ dy/dx$

Multiply both sides by $\sqrt{5}$

$3(2 + dy/dx) = 50\sqrt{5} + 100\sqrt{5}\ dy/dx$

Expand

$6 + 3\ dy/dx = 50\sqrt{5} + 100\sqrt{5}\ dy/dx$

Collect dy/dx terms

$3\ dy/dx - 100\sqrt{5}\ dy/dx = 50\sqrt{5} - 6$

$dy/dx(3 - 100\sqrt{5}) = 50\sqrt{5} - 6$

$dy/dx = (50\sqrt{5} - 6)/(3 - 100\sqrt{5})$

2.(a) A student is required to answer fourteen questions out of eighteen questions in a test. How many choices has she?

Number of choices

$= 18C14$

$18C14 = 18C4$

$18C4 = (18 \times 17 \times 16 \times 15)/(4 \times 3 \times 2 \times 1)$

$18C4 = 3060$

(b) If she must answer the first ten questions how many choices has she?

Remaining questions to choose from

$= 18 - 10 = 8$

Remaining questions to answer

$= 14 - 10 = 4$

Number of choices

$= 8C4$

$8C4 = (8 \times 7 \times 6 \times 5)/(4 \times 3 \times 2 \times 1)$

$8C4 = 70$

3.(a) Using a calculator, find the value of $\tan 36°48'$.

$36°48' = 36.8°$

$\tan 36.8° \approx 0.748$

(b) Using a calculator, find the value of $\ln(\sin^3 2)$.

$\sin 2 \approx 0.9093$

$\sin^3 2 = (0.9093)^3$

$\sin^3 2 \approx 0.751$

$\ln(0.751) \approx -0.286$

4.(a) Mention four important steps to be followed when formulating a system of linear constraints from a word problem.

One important step is to define the decision variables clearly so that each unknown quantity in the problem is represented correctly.

Another step is to translate the conditions of the word problem into mathematical inequalities using the defined variables.

A further step is to identify all restrictions such as non negativity conditions that must be satisfied by the variables.

The final step is to check the formulated constraints to ensure they accurately represent the original problem situation.

(b) Sketch the following system of linear inequalities: $x \geq 0$, $y \geq 0$, $x + 3y \leq 6$.

The inequality $x \geq 0$ represents the region to the right of the y-axis.
The inequality $y \geq 0$ represents the region above the x-axis.

For $x + 3y = 6$
When $x = 0$, $y = 2$
When $y = 0$, $x = 6$

The feasible region is the triangular area bounded by the x-axis, y-axis, and the line $x + 3y = 6$ including the boundaries.

5.(a) Using the following information:
Q is the midpoint of segment PR
$PT \cong RS$

TQ ≅ SQ

Prove that ΔPQT ≅ ΔRQS.

Since Q is the midpoint of PR, PQ = QR.

Given PT ≅ RS, corresponding sides are equal.

Given TQ ≅ SQ, another pair of corresponding sides are equal.

Therefore PQ = QR, PT = RS, and TQ = SQ.

By SSS congruence criterion, ΔPQT ≅ ΔRQS.

6. List down four importance of teaching mathematics in schools.

Teaching mathematics develops logical thinking and problem solving skills which are essential in everyday life.

Mathematics equips learners with numerical skills required for careers in science, technology, and commerce.

It helps learners understand and interpret data, measurements, and quantitative information in society.

Mathematics promotes accuracy, reasoning, and discipline in thinking.

7. Give three reasons why a table of specification is very important to a teacher in constructing mathematics test.

A table of specification ensures balanced coverage of syllabus content so that all important topics are tested fairly.

It helps the teacher align test items with instructional objectives and cognitive levels.

It improves the validity and reliability of the test by guiding proper distribution of questions.

8. Outline three advantages of using a textbook as one of the curriculum materials in learning mathematics.

A textbook provides structured and organized content that guides both teaching and learning.

It offers worked examples and exercises that help learners practice and reinforce concepts.

Textbooks serve as a reliable reference for revision and independent study.

9. Identify six criteria of a well stated objective.

A well stated objective should be clear and specific to avoid ambiguity.

It should be measurable so that achievement can be assessed.

The objective should be achievable within the given learning conditions.

It should be learner centered, focusing on what the learner will be able to do.

The objective should specify observable behavior.

It should be relevant to the curriculum and lesson content.

10. Briefly explain two principles that you could use to promote mathematics learning among the learners.

One principle is active participation, where learners are involved in problem solving, discussions, and activities, enabling deeper understanding.

Another principle is progression from simple to complex concepts, which helps learners build confidence and understanding step by step.

11.(a) Using integration, show that the volume of a sphere is $4/3\ \pi r^3$.

Consider a sphere of radius r.
The equation of the generating semicircle is $y^2 = r^2 - x^2$.
The volume by rotation about the x-axis is
$V = \pi \int$ from $-r$ to r of $y^2\ dx$
$V = \pi \int$ from $-r$ to r of $(r^2 - x^2)\ dx$
$V = \pi [\int r^2\ dx - \int x^2\ dx]$
$V = \pi [r^2x - x^3/3]$ from $-r$ to r
At x = r, $r^2r - r^3/3 = 2r^3/3$
At x = $-r$, $-r^3 + r^3/3 = -2r^3/3$
$V = \pi [2r^3/3 - (-2r^3/3)]$
$V = 4/3\ \pi r^3$

(b) Integrate 4x divided by $(2x^2 + x - 3)$ with respect to x.

Let $D = 2x^2 + x - 3$
$dD/dx = 4x + 1$
Rewrite 4x as $(4x + 1) - 1$

$\int 4x / (2x^2 + x - 3)\ dx$
$= \int (4x + 1)/(2x^2 + x - 3)\ dx - \int 1/(2x^2 + x - 3)\ dx$

$\int (4x + 1)/(2x^2 + x - 3)\ dx = \ln|2x^2 + x - 3|$

Factor denominator

$2x^2 + x - 3 = (2x - 3)(x + 1)$

$1/(2x^2 + x - 3) = A/(2x - 3) + B/(x + 1)$

$1 = A(x + 1) + B(2x - 3)$

Comparing coefficients

$A = 2/5$

$B = -1/5$

$\int 1/(2x^2 + x - 3)\, dx$

$= 2/5 \ln|2x - 3| - 1/5 \ln|x + 1|$

Final answer

$\ln|2x^2 + x - 3| - 2/5 \ln|2x - 3| + 1/5 \ln|x + 1| + C$

12.(a) Find the derivative of the given f(x) = uv divided by w.

Using the quotient rule,

$f = (uv)/w$

$df/dx = [\, w\, d(uv)/dx - uv\, dw/dx \,] / w^2$

Since $d(uv)/dx = u\, dv/dx + v\, du/dx$,

$df/dx = [\, w(u\, dv/dx + v\, du/dx) - uv\, dw/dx \,] / w^2$

(b) From the first principle, find the derivative of $(x + 1)/(x - 1)$ with respect to x.

Let $f(x) = (x + 1)/(x - 1)$

f′(x) = limit as h approaches 0 of

[(x + h + 1)/(x + h − 1) − (x + 1)/(x − 1)] / h

Combine fractions

Numerator = (x + h + 1)(x − 1) − (x + 1)(x + h − 1)

Expand

= (x² − x + hx − h + x − 1 + h) − (x² + hx − x + x + h − 1)

Simplify

= −2h

f′(x) = −2h / [h(x + h − 1)(x − 1)]

Cancel h

f′(x) = −2 / [(x + h − 1)(x − 1)]

Let h approach 0

f′(x) = −2 / (x − 1)²

13.(a) Using laws of algebra simplify the proposition ¬(p ∧ q) ∧ (¬p ∨ q).

¬(p ∧ q) = ¬p ∨ ¬q

Expression becomes

(¬p ∨ ¬q) ∧ (¬p ∨ q)

Apply distributive law

= ¬p ∨ (¬q ∧ q)

Since ¬q ∧ q is false,

Final result is ¬p

(b) Verify that (p ∧ q) → (p ∨ q) is a tautology.

If p ∧ q is true, both p and q are true, so p ∨ q is true.
If p ∧ q is false, the implication is true by definition.
Therefore the statement is true in all cases and is a tautology.

(c) Draw an electrical network of the proposition (p ∧ q) ∨ (p ∧ r).

The network consists of two parallel branches.
One branch has switch p in series with switch q.
The second branch has switch p in series with switch r.
Current flows if either branch is complete.

14.(a) Find the equation of the tangent line which touches ellipse at the point (−2, 1).

For an ellipse, the tangent at (x_1, y_1) is
$xx_1/a^2 + yy_1/b^2 = 1$

Substitute $x_1 = -2$ and $y_1 = 1$

$-2x/a^2 + y/b^2 = 1$

(b) Show that $16x^2 + 25y^2 + 96x - 50y = 231$ is an equation of ellipse. Hence, find its centre.

Group terms

$16(x^2 + 6x) + 25(y^2 - 2y) = 231$

Complete the squares

$x^2 + 6x = (x + 3)^2 - 9$

$y^2 - 2y = (y - 1)^2 - 1$

Substitute

$16[(x + 3)^2 - 9] + 25[(y - 1)^2 - 1] = 231$

Simplify

$16(x + 3)^2 + 25(y - 1)^2 = 400$

Divide by 400

$(x + 3)^2/25 + (y - 1)^2/16 = 1$

This is an ellipse with centre (−3, 1).

15.(a) Write three specific objectives of the subtopic domain and range.

One objective is to enable learners to identify the domain of a function correctly by analyzing algebraic expressions and graphs, ensuring valid input values are recognized.

Another objective is to help learners determine the range of a function by examining outputs and graphical behavior, strengthening understanding of functional relationships.

A further objective is to develop the ability to apply domain and range concepts in solving real life problems involving relations and mappings.

(b) Write its main topic and the competence of the topic.

The main topic is Functions.
The competence developed is the ability to analyze and use functions correctly by identifying their domains and ranges in mathematical and real life contexts.

(c) Describe a good introduction of the subtopic while teaching.

A good introduction begins with familiar real life examples such as assigning students to admission numbers or matching people with phone numbers. These examples naturally introduce inputs and outputs. The teacher then links these ideas to functions and formally introduces domain as possible inputs and range as resulting outputs.

16. Suppose you are in a panel for preparation of a mathematics textbook for ordinary level mathematics. Describe five characteristics of a textbook with high quality.

A high quality textbook should strictly follow the syllabus and present mathematically correct content to ensure curriculum alignment.

It should use clear language and logical progression so learners can easily follow concepts without confusion.

Worked examples should be well explained and followed by graded exercises that build confidence and skill.

Accurate diagrams and illustrations should be used to support understanding of abstract ideas.

The textbook should promote problem solving and critical thinking through application based questions.

17. Evaluate five challenges that teachers meet while using the basic mathematics syllabus for secondary schools in Tanzania. Give five suggestions to remedy the situation.

One major challenge is large class size, which limits individual learner support and interaction.

Another challenge is shortage of textbooks and teaching aids, reducing effective syllabus coverage.

Weak mathematical foundation among learners makes teaching advanced topics difficult.

Limited instructional time makes it hard to complete the wide syllabus.

Inadequate in-service training leaves some teachers without updated teaching methods.

Reducing class size or using group teaching can improve learner participation.

Providing adequate textbooks and teaching aids enhances learning effectiveness.

Introducing remedial classes can support learners with weak foundations.

Improving time allocation helps teachers manage syllabus coverage.

Regular professional development equips teachers with effective teaching strategies.

18. Prepare two multiple-choice test items with four distracters from each of the following themes.

(a) Domain and Range

Question 1

Find the domain of $f(x) = 1/(x - 2)$.

A. All real numbers

B. $x \neq 2$

C. $x > 2$

D. $x < 2$

Question 2

The range of $f(x) = x^2$ is

A. All real numbers

B. $y \geq 0$

C. $y \leq 0$

D. $y \neq 0$

(b) Tangent properties

Question 1

A tangent to a circle is a line which

A. Cuts the circle at two points

B. Touches the circle at one point

C. Passes through the center

D. Is parallel to the radius

Question 2

The angle between a tangent and the radius at the point of contact is

A. Acute

B. Obtuse

C. Right angle

D. Straight angle

(c) Chord properties of a circle

Question 1

Equal chords in the same circle are

A. At different distances from the center

B. At equal distances from the center

C. Always diameters

D. Always perpendicular

Question 2

A chord passing through the center of a circle is called

A. Radius

B. Tangent

C. Diameter

D. Secant