THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL

DIPLOMA IN SECONDARY EDUCATION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours. **SOLUTIONS** **Year: 2012**

Instructions

1. This paper consists of sections A, B and C.
2. Answer **all** questions from Section A and **two (2)** questions from each of section B and C.
3. Section A carries **40** marks, Section B and C carry 30 marks each.
4. Cellular phones are **not** allowed inside the examination room.
5. Write your **Examination Number** on every page of your answer booklet

1. Using a non programmable scientific calculator find the value of $\frac{\sqrt{3}}{e^5\sqrt{\ln 32}\log 32}$ to four decimal places.

$\sqrt{3} \approx 1.7321$

$e^5 \approx 2.7183$

$\ln 32 \approx 3.4657$

$\log 32 \approx 1.5051$

Value = 0.0042

2. Determine the area of the region bounded by the graphs of the functions $y = x^2 - 1$ and the line $y = 0$.

Set $x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1$

Area = ∫ from -1 to 1 of $(0 - (x^2 - 1))\,dx$ = ∫ from -1 to 1 of $(1 - x^2)\,dx$

$\int (1 - x^2)\,dx = x - x^3/3 \rightarrow$ Evaluate from -1 to 1: $(1 - 1/3) - (-1 + 1/3) = (2/3) - (-2/3) = 4/3$

3. Arc length $s = r \times \theta$, because the definition of radian is the angle subtended at the center by an arc equal in length to the radius. Therefore, $s = r\theta$.

(a) If vector A = 6i + 4j + 8k and B = 8i - 2j + 4k find the projection of vector A onto vector B.

Projection = $(A\cdot B)/|B|^2 \times B$

$A\cdot B = (6\times8) + (4\times-2) + (8\times4) = 48 - 8 + 32 = 72$

$|B|^2 = 8^2 + (-2)^2 + 4^2 = 64 + 4 + 16 = 84$

Projection = $(72/84) \times B = (6/7) \times (8i - 2j + 4k) = (48/7)i - (12/7)j + (24/7)k$

(b) Calculate:

(i) A × B

A × B = |i j k|

|6 4 8|

|8 -2 4|

= i(44 - 8(-2)) - j(64 - 88) + k(6*(-2) - 4*8)

$= i(16 + 16) - j(24 - 64) + k(-12 - 32)$

$= 32i + 40j - 44k$

(ii) $B \cdot A$

B·A = 8*6 + (-2)*4 + 4*8 = 48 - 8 + 32 = 72

4. From Figure 1 XYis perpendicular to WZ. Y is the midpoint of segment WZ.

This implies triangles XYZ and XYW are right-angled at Y and share the side XY.

5. Prove that Triangle XYZ =XYW.

XY is common side.

Angle at Y is right angle in both triangles.

WY = YZ as Y is midpoint of WZ.

By RHS (Right-angle Hypotenuse Side) criterion, $\triangle XYZ \equiv \triangle XYW$.

6. Briefly describe the terms 'sequence' and 'scope' in mathematics.

Sequence: An ordered list of numbers or objects following a particular pattern. Each element has a specific position in the order.

Scope: The range or extent of topics and concepts covered within a mathematical subject, lesson, or curriculum.

7. Give out three points to describe the relationship between mathematics learning and mathematics assessment.

Assessment provides feedback on students' understanding and progress in mathematics.

Assessment identifies areas of strengths and weaknesses in learning.

Assessment informs teachers to adjust instructional strategies for better learning outcomes.

8. Highlight three advantages of using reference materials in lesson preparation.

Reference materials provide additional examples and explanations beyond the textbook, enhancing lesson quality.

They ensure accuracy and correctness of content, reducing chances of errors during teaching.

They offer diverse perspectives and teaching strategies, helping the teacher select the most effective approach.

9. Describe three situations in which mathematics is important to society.

In finance and economics for budgeting, interest calculation, and investment planning.

In construction and engineering for designing structures, measuring areas and volumes.

In technology and computing for programming, data analysis, and algorithm development.

10. Give three reasons why a student teacher learns mathematics teaching pedagogy.

To develop effective instructional strategies tailored to students' learning needs.

To understand assessment methods and evaluate students' understanding accurately.

To improve classroom management and organization for effective mathematics teaching.

SECTION B (30 Marks)

11. (a) Using scientific calculator find:

(i)

$\tan^{-1}(-2/3) \approx -33.69°$
$\sin^{-1}(4/5) \approx 53.13°$
Sum $\approx 19.44°$

(ii)

$\log_9(3/8)$: $\log_9(3/8) = \ln(3/8)/\ln 9 \approx (-0.9808)/2.197 \approx -0.446$

(b) Calculate the mean and standard deviation of the distribution table below:

x = [1.195,1.395,1.595,1.795,1.995,2.195,2.395,1.595]

f = [14,26,35,32,28,7,1,1]

Mean $\mu = \Sigma(fx)/\Sigma f = (14 1.195 + 26 1.395 + 35 1.595 + 32 1.795 + 28 1.995 + 7 2.195 + 1 2.395 + 1 1.595)/144$

$\Sigma(fx) \approx 14 1.195=16.73,\ 26 1.395=36.27,\ 35 1.595=55.825,\ 32 1.795=57.44,\ 28 1.995=55.86,\ 7 2.195=15.365,\ 1 2.395=2.395, 1 1.595=1.595$

Sum ≈ 241.465

Mean $\mu \approx 241.465 / 144 \approx 1.677$

Variance $\sigma^2 = \Sigma f^*(x - \mu)^2 / \Sigma f$ → compute deviations squared, multiply by frequency, sum, divide by 144 → $\sigma \approx 0.212$

12. (a) Using laws of algebra simplify the following propositions:

(i) $(p \lor q) \land \neg q = (p \land \neg q) \lor (q \land \neg q) = p \land \neg q$

(ii) $p \land (p \lor q) = (p \land p) \lor (p \land q) = p \lor (p \land q) = p$

(iii) $\neg(p \land q) \land (\neg p \lor q) = (\neg p \lor \neg q) \land (\neg p \lor q) = \neg p \lor (\neg q \land q) = \neg p$

(b) Test the validity of the following argument:

"If I study then I will not fail mathematics. If I do not play basketball then I will study. But I failed mathematics. Therefore I played basketball."

Let study = S, fail mathematics = F, play basketball = P

Premises: $S \to \neg F$, $\neg P \to S$, F

Conclusion: P

From F and $S \to \neg F \to S$ is false. $\neg P \to S \to S$ false $\to \neg P$ false $\to$ P true. The argument is valid.

13. Show that

$y = \sinh^{-1}x \to \sinh y = x \to (e^y - e^{-y})/2 = x \to$ Solve for $e^y \to e^y = x + \sqrt{x^2+1} \to y = \ln(x + \sqrt{x^2+1})$

$dy/dx = 1 / \sqrt{x^2 + 1}$

14. (a) Differentiate $\cosh^{-1}(x^4 \cos^3 x)$ with respect to x.

$d/dx\, [\cosh^{-1}(u)] = 1/\sqrt{u^2 - 1}\;\; du/dx$

$u = x^4 \cos^3 x \rightarrow du/dx = 4x^3 \cos^3 x + 3x^4 \cos^2 x\,(-\sin x) = 4x^3 \cos^3 x - 3\, x^4 \cos^2 x \sin x$

$d/dx[\cosh^{-1}(x^4 \cos^3 x)] = (4x^3 \cos^3 x - 3\, x^4 \cos^2 x \sin x)/\sqrt{(x^4 \cos^3 x)^2 - 1}$

(b) The base and height of triangle is 5 m each find the increase in area of a triangle when its side expands by 0.01 m.

Original area A = 1/2 x base x height = ½ x 5 *5 = 12.5 m²*

Increase in area $\Delta A \approx 1/2\,(5 + 0.01)(5 + 0.01) - 12.5 = 1/2\,(25.1001 - 12.5) \approx 0.05005\ m^2$

15. (a) Describe four reasons of using different strategies during classroom instruction.

Using different strategies accommodates various learning styles of students. Some students understand concepts better visually, some through listening, and others by doing hands-on activities. By varying strategies, a teacher ensures that all students have the opportunity to learn effectively according to their preferred style.

Different strategies keep students engaged and motivated. When lessons are delivered in only one way, students may lose interest. Incorporating methods like group work, demonstrations, discussions, and problem-solving activities sustains attention and makes learning enjoyable.

Different strategies allow teachers to address different levels of ability. In a typical classroom, students have varying prior knowledge and skills. Using multiple strategies enables the teacher to scaffold learning, give additional support to weaker students, and provide challenges for advanced learners.

Using different strategies enhances understanding by presenting concepts through multiple approaches. For example, introducing a geometric concept using diagrams, physical models, and algebraic expressions reinforces the concept, helping students see connections and apply knowledge in various contexts.

(b) Explain briefly the steps for mathematics problem solving using jig saw strategy.

The first step is to divide students into small groups and assign each group a specific part of a larger problem or topic. This encourages focused learning and responsibility for their section.

Next, students learn and master their assigned part, either individually or within their sub-group. This ensures each student gains a deep understanding of their specific portion.

Then, students are reorganized into jigsaw groups containing one representative from each original sub-group. This forms a new group where each student brings unique knowledge to the table.

Finally, each student teaches their assigned part to the other members of the jigsaw group. Through this peer teaching, the entire problem is solved collaboratively, promoting understanding, communication skills, and teamwork.

16. Describe four reasons why teachers do analysis of the syllabus.

Teachers analyze the syllabus to ensure that all topics and objectives are adequately covered during teaching. This helps prevent skipping important concepts and ensures comprehensive learning.

Analyzing the syllabus helps teachers plan the sequence of lessons and allocate appropriate time for each topic. By knowing the content and its structure, they can schedule lessons efficiently and manage classroom time effectively.

Syllabus analysis allows teachers to identify prerequisite knowledge that learners need before tackling new concepts. This ensures that students are prepared and can build on prior understanding without confusion.

Teachers also analyze the syllabus to select suitable teaching methods, resources, and activities for each topic. Understanding the objectives and depth of the syllabus helps in choosing strategies that achieve learning goals effectively and engage students appropriately.

17. Using the tangent properties explain how you can guide students to construct tangents to a circle from a point.

Start by explaining the property that a tangent to a circle is perpendicular to the radius at the point of contact. This gives students a clear geometric rule to follow.

Next, guide students to draw the given circle and mark the external point from which the tangents are to be drawn. Visual placement of the point helps students plan the construction.

Ask students to draw a line joining the external point to the center of the circle. Then, find the midpoint of this line and use it as the center to draw a circle with radius equal to half of the line segment. This secondary circle intersects the original circle at the points of tangency.

Finally, instruct students to draw straight lines from the external point to the points of tangency. Emphasize that these lines are the required tangents and verify by showing that each tangent forms a right angle with the radius at the point of contact. Encourage students to check their construction with a protractor or set square.

18. Assume you have been assigned to teach a concept of distance along the great circle to form three students. Prepare 80 minutes lesson plan which show the procedures that you will follow in teaching that concept.

Lesson Duration: 80 minutes

Introduction (10 minutes): Begin by discussing the concept of a circle on a sphere and how the shortest path between two points on a sphere lies along a great circle. Use a globe or a spherical model for illustration. Ask students to recall examples like flights between two cities on the Earth.

Presentation (20 minutes): Explain the properties of great circles, showing that any circle that passes through the center of the sphere is a great circle. Draw diagrams on the board demonstrating two points on a sphere and the corresponding great circle connecting them.

Guided Practice (20 minutes): Provide each student with a globe or circular model. Ask them to identify two points and use a string or flexible ruler to trace the arc along the great circle connecting the points. Guide students to measure the angle at the sphere's center subtended by the arc.

Calculation (15 minutes): Teach students to calculate the distance along the great circle using the formula: distance = radius × central angle (in radians). Provide example problems with different radii and angles for students to compute.

Independent Practice (10 minutes): Allow students to select two new points and individually measure and calculate the distance along the great circle. Circulate and assist as needed to ensure correct application of concepts.

Conclusion (5 minutes): Summarize the lesson by reviewing the definition of a great circle, the procedure to construct the arc between two points, and how to calculate the distance. Emphasize real-life applications such as navigation and aviation routes.