THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL

DIPLOMA IN SECONDARY EDUCATION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours. **SOLUTIONS** **Year: 2018**

Instructions

1. This paper consists of sections A, B and C.

2. Answer **all** questions from Section A and **two (2)** questions from each of section B and C.

3. Section A carries **40** marks, Section B and C carry 30 marks each.

4. Cellular phones are **not** allowed inside the examination room.

5. Write your **Examination Number** on every page of your answer booklet

1. Define the following terms as used in mathematics teaching methods:

   (a) Curriculum materials

   Curriculum materials are resources and tools provided to support the teaching and learning of mathematics, including textbooks, workbooks, teaching aids, charts, and digital resources that align with the curriculum objectives.

   (b) Mathematics syllabus

   A mathematics syllabus is a structured outline of topics, concepts, skills, and learning objectives that students are expected to study and achieve over a specified period, such as a term or academic year.

   (c) Lesson plan

   A lesson plan is a detailed guide prepared by a teacher for a specific lesson, outlining objectives, teaching methods, learning activities, resources, and assessment methods to ensure effective delivery of the lesson.

   (d) Assessment

   Assessment is the process of evaluating students' understanding, skills, and performance in mathematics, which can be done through tests, quizzes, assignments, observations, or practical tasks.

2. Differentiate between the following:

   (a) Mathematics text book and mathematics Reference book

   A mathematics textbook is designed to guide students through the syllabus sequentially, providing explanations, examples, exercises, and practice questions. A mathematics reference book is a resource used for additional information, deeper understanding, or clarification on specific topics, but it is not structured to follow the syllabus for teaching.

   (b) Mathematics syllabus and mathematics teacher's guide book

   A mathematics syllabus outlines what students are expected to learn, including topics, objectives, and assessment criteria. A mathematics teacher's guide book provides instructions, teaching strategies, lesson plans, and explanations to help teachers effectively deliver the syllabus content.

3. (a) Write the principle of permutation as used in probability.

   The principle of permutation states that the number of ways to arrange 'n' distinct objects in order is n factorial (n!). For arranging 'r' objects from 'n' distinct objects, the number of permutations is n! / (n-r)!.

(b) Ten candidates are contesting for presidency. How many ways are there of predicting the first three positions?

Number of ways = 10P3 = 10 × 9 × 8 = 720 ways.

4. During micro-teaching session, a mathematics teacher who was teaching about 3-Dimensional geometry in Form Four asked students to mention any three prism geometrical objects they know. The following were the responses of students A and B.

Student A	Student B
Rectangular	Triangular
Cylinder	Cone
Cube	Pyramid

5. The teacher accepted all responses from students A and B as correct responses. Assume you were an observer in the micro-teaching, what would have been your reaction to the teacher's comment?

   My reaction would be that the teacher partially accepted correct responses. Rectangular prism, triangular prism, and cube are correct prism shapes. Cylinder, cone, and pyramid are not prisms; a cylinder is a curved surface solid, and cone and pyramid are pyramidal solids. The teacher should provide clarification to distinguish prisms from other 3-D objects.

6. Indicate the rules used to simplify the following proposition;

   $P \wedge (\neg P \vee \neg Q) \equiv P \wedge (\neg P \vee \neg Q) \rightarrow$ given

   $\equiv (P \wedge \neg P) \vee (P \wedge \neg Q) \rightarrow$ distributive law

   $\equiv \emptyset \vee (P \wedge \neg Q) \rightarrow$ complement law

   $\equiv P \wedge \neg Q \rightarrow$ identity law

   $\equiv P \wedge (\neg P \vee \neg Q) \equiv P \rightarrow$ simplification and equivalence

7. Find two real numbers x and y such that $(2+i)x + (3-2i)y = -1 - 4i$.

   Separate into real and imaginary parts:

   Real: $2x + 3y = -1$

   Imaginary: $x - 2y = -4$

   Solve simultaneously:

From imaginary: $x = -4 + 2y$

Substitute into real: $2(-4 + 2y) + 3y = -1 \rightarrow -8 + 4y + 3y = -1 \rightarrow 7y = 7 \rightarrow y = 1$

Then $x = -4 + 2(1) = -2$

Solution: $x = -2, y = 1$

8. Differentiate the function $f(x) = \cos(x^2 + 2x + 1)$.

   $f'(x) = -\sin(x^2 + 2x + 1) \times d/dx(x^2 + 2x + 1)$

   $d/dx(x^2 + 2x + 1) = 2x + 2$

   Therefore, $f'(x) = -(2x + 2) \sin(x^2 + 2x + 1)$

9. Sketch a diagram of a square pyramid and mention the total number of:

   (a) the faces → 5 faces (1 square base + 4 triangular sides)

   (b) the vertices → 5 vertices (4 base corners + apex)

   (c) the edges → 8 edges (4 base edges + 4 side edges connecting apex)

10. List four characteristics of a learner centered teaching method.

    Learner-centered teaching encourages active participation by students in learning activities.

    It focuses on the needs, interests, and prior knowledge of learners.

    It promotes critical thinking, problem-solving, and independent learning skills.

    It involves continuous feedback, reflection, and adaptation to support learner understanding.

10. Find the perpendicular distance from point (10, -11) to the line passing through points (2, -1) and (1, 1).

    Slope of line: $m = (1 - (-1)) / (1 - 2) = 2 / -1 = -2$

    Equation of line: $y - (-1) = -2(x - 2) \rightarrow y + 1 = -2x + 4 \rightarrow 2x + y - 3 = 0$

    Distance $d = |2(10) + 1(-11) - 3| / \sqrt{(2^2 + 1^2)} = |20 -11 -3| / \sqrt{5} = 6 / \sqrt{5} \approx 2.683$ units

11. (a) Solve the following equation for the value of x where $0° \leq x \leq 180°$:

    (i) $\cos(x + 30°) - \cos(x + 90°) = 1/2$

    Use $\cos A - \cos B = -2 \sin((A+B)/2) \sin((A-B)/2)$

    $A = x + 30°, B = x + 90°$

    $-2 \sin((2x + 120°)/2) \sin((-60°)/2) = 1/2 \rightarrow -2 \sin(x + 60°) \sin(-30°) = 1/2$

    $-2 \sin(x + 60°)(-1/2) = 1/2 \rightarrow \sin(x + 60°) = 1/2 \rightarrow x + 60° = 30°, 150° \rightarrow x = -30°, 90°$ → valid: $x = 90°$

(ii) $\cos(x + 30°)\cos(x - 30°) = 1/2$

Use identity: $\cos(A)\cos(B) = 1/2[\cos(A-B) + \cos(A+B)]$

$1/2[\cos(60°) + \cos(2x)] = 1/2 \rightarrow 1/2[1/2 + \cos(2x)] = 1/2 \rightarrow 1/4 + 1/2\cos(2x) = 1/2 \rightarrow \cos(2x) = 1/2 \rightarrow$

$2x = 60°, 300° \rightarrow x = 30°, 150°$

(b) Use t-formula to solve the equation $\sin\theta + 2\cos\theta = 1$

$t = \tan(\theta/2)$

$\sin\theta = 2t / (1 + t^2)$, $\cos\theta = (1 - t^2) / (1 + t^2)$

$2t / (1 + t^2) + 2(1 - t^2)/(1 + t^2) = 1 \rightarrow (2t + 2 - 2t^2)/(1 + t^2) = 1 \rightarrow -2t^2 + 2t + 2 = 1 + t^2 \rightarrow -3t^2 + 2t + 1 = 0$

$\rightarrow 3t^2 - 2t - 1 = 0$

$t = [2 \pm \sqrt{(4 + 12)}]/6 = [2 \pm 4]/6 \rightarrow t = 1, t = -1/3$

$\theta = 2\arctan(1) = 90°$, $\theta = 2\arctan(-1/3) \approx -36.87° \rightarrow$ add 180° for positive: $\theta \approx 143.13°$

Solutions: $\theta = 90°, 143.13°$

12. (a) If A(1,1,3) and B(4,5,8) find displacement vector AB in terms of i, j, k

AB = B - A = (4-1)i + (5-1)j + (8-3)k = 3i + 4j + 5k

In xyz plane, vector starts at (1,1,3) and ends at (4,5,8)

(b) Particle $r(t) = 3t^2i + 2tj - e^{\wedge}tk$

Velocity $v = dr/dt = 6t\ i + 2\ j - e^{\wedge}t\ k$

At t=3s: $v = 18i + 2j - 20.0855\ k$

Acceleration $a = dv/dt = 6i + 0\ j - e^{\wedge}t\ k$

At t=3s: $a = 6i + 0\ j - 20.0855\ k$

13. (a) Use Taylor's series to expand $\sin(\pi/6 + k)$ to k^2

$\sin(\pi/6 + k) = \sin(\pi/6) + k\cos(\pi/6) - k^2/2\sin(\pi/6) + ...$

$\sin(\pi/6) = 1/2$, $\cos(\pi/6) = \sqrt{3}/2 \rightarrow \sin(\pi/6 + k) \approx 1/2 + (\sqrt{3}/2)k - (1/4)k^2$

(b) $y = x^3 - 2x^2 + x + 1$

$dy/dx = 3x^2 - 4x + 1 \rightarrow$ set $dy/dx = 0 \rightarrow 3x^2 - 4x + 1 = 0 \rightarrow (3x - 1)(x - 1) = 0 \rightarrow x = 1/3, 1$

$d^2y/dx^2 = 6x - 4 \rightarrow$ at x=1/3: $-2 \rightarrow$ max, at x=1: $2 \rightarrow$ min

Turning points: (1/3, y(1/3)), (1, y(1))

Sketch curve: one root only at intersection with x-axis

14. Explain five factors which a teacher has to consider during construction of mathematics test.

The objectives of the test must align with the curriculum and intended learning outcomes.

The level of difficulty should match the learners' abilities and grade level.

The test should cover a representative sample of the syllabus content.

Questions should be clear, unambiguous, and free from errors.

Time allocation and format should be suitable for the number of questions and assessment purpose.

15. (a) Using the following sketched cuboid, prepare part of lesson development of a lesson plan for teaching "how to locate and name an angle between a line and plane of a cuboid".
Begin by introducing basic terminology:

line, plane, and angle. Show the cuboid and identify the chosen line and plane. Demonstrate how to visualize perpendiculars and measure the angle.

Guide learners through examples using coordinates or edges to locate the angle. Include guided practice and questions for students to attempt.

(b) By giving a reason, identify two prerequisite concepts knowledge that learners need to have in order to understand the procedures for calculating an angle between a line and a plane of a three dimensional figure.
Learners should understand the concept of a plane and a line in three dimensions to identify geometric elements.
Learners should know how to calculate angles between vectors or line segments using trigonometric ratios or dot product.

16. The use of lesson plan plays an important role in teaching and learning mathematics. Justify this statement by giving five points.
A lesson plan ensures that the teacher covers all planned topics systematically and efficiently.

It helps the teacher organize resources and teaching aids in advance.

It provides clear objectives and learning outcomes for both teacher and learners.

It facilitates assessment planning and monitoring of student progress.
It serves as a guide for reflection and improvement of teaching practices.