THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL

DIPLOMA IN SECONDARY EDUCATION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours. **SOLUTIONS** **Year: 2019**

Instructions

1. This paper consists of sections A, B and C.
2. Answer **all** questions from Section A and **two (2)** questions from each of section B and C.
3. Section A carries **40** marks, Section B and C carry 30 marks each.
4. Cellular phones are **not** allowed inside the examination room.
5. Write your **Examination Number** on every page of your answer booklet

1. Identify four great Mathematicians in mathematics history and briefly explain the contribution of each one.

Pythagoras contributed greatly through his discovery of the Pythagoras theorem which relates the sides of a right-angled triangle as $a^2 + b^2 = c^2$. This theorem is widely used in geometry and trigonometry.

Isaac Newton contributed to the development of calculus which is used to study rates of change and motion. His mathematical formulations also laid the foundation for classical mechanics.

Euclid is known as the father of geometry. His book “Elements” systematically organized geometric knowledge and introduced axioms and postulates that are still used today.

Carl Friedrich Gauss contributed to number theory and algebra. He developed the Gaussian distribution and made significant progress in complex numbers and modular arithmetic.

2. Briefly explain Zoltan Dienes view in learning of mathematics.

Zoltan Dienes believed that mathematics learning should progress through stages from concrete to abstract. He emphasized the use of manipulatives and structured experiences to help learners understand concepts.

He proposed that students should first explore mathematical ideas through play, then generalize through structured activities, and finally represent ideas symbolically. This approach helps learners build a deeper understanding rather than memorizing formulas.

3. Given the statement that "If I am under nine years old, then I will go to school." Write its Converse, Inverse and Contrapositive.

Original statement: If I am under nine years old, then I will go to school.

Converse: If I go to school, then I am under nine years old.

Inverse: If I am not under nine years old, then I will not go to school.

Contrapositive: If I do not go to school, then I am not under nine years old.

4. In the figure, the line segment BD is perpendicular to AC, D is the midpoint of AB and AC respectively. Prove that triangles ABE and ACD are congruent.

Since BD is perpendicular to AC, $\angle BDA = \angle CDA = 90°$.
D is the midpoint of both AB and AC, hence AD is common in both triangles.

By the RHS congruence rule, triangle ABE ≅ triangle ACD because they have equal hypotenuse and one side equal and right angles equal.

5. Mention four curriculum materials applied in teaching and learning of mathematics subject.

Textbooks are important materials that provide detailed explanations and exercises for both teachers and students.

Mathematics teaching aids such as geometric instruments, number lines, and charts help to visualize abstract concepts.

Teacher's guide books help teachers to plan lessons effectively and follow the approved curriculum.

Mathematics software and audio-visual materials like projectors or simulations enhance interactive learning in class.

6. A die is thrown once and the sum of the numbers appearing is observed to be 8. What is the probability that number 6 appearing is at least one?

Possible outcomes for sum 8: (2,6), (3,5), (4,4), (5,3), (6,2).
Favorable outcomes with at least one 6: (2,6), (6,2).
Number of favorable outcomes = 2.
Total possible outcomes = 5.
Probability = 2/5.

7. Find the equation of the tangent to the ellipse $x^2/a^2 + y^2/b^2 = 1$ at the point (a cosθ, b sinθ).

Equation of tangent: $(x\cos\theta)/a + (y\sin\theta)/b = 1$.

8. Consider a point P which divides the line joining points (4,2,2) and (10,6,4) in the ratio (1:1) internally. Find the coordinates of point P.

Using the section formula,
$P(x, y, z) = [(m_2x_1 + m_1x_2)/(m_1 + m_2), (m_2y_1 + m_1y_2)/(m_1 + m_2), (m_2z_1 + m_1z_2)/(m_1 + m_2)]$
$= [(1x4 + 1x10)/(1 + 1), (1x2 + 1x6)/(1 + 1), (1x2 + 1x4)/(1 + 1)]$
$= (7, 4, 3)$.

9. Differentiate $\cosh(\sqrt{(x^2 + 1)})$ with respect to x.

Let $y = \cosh(\sqrt{(x^2 + 1)})$.

$dy/dx = \sinh(\sqrt{(x^2 + 1)}) \times (1 / (2\sqrt{(x^2 + 1)})) \times 2x$

$dy/dx = x \times \sinh(\sqrt{(x^2 + 1)}) / \sqrt{(x^2 + 1)}$.

10. By using the cross product of vectors, find the angle between vector $a = 2i + j - 2k$ and $b = i - 2j + k$.

$a \cdot b = (2x1) + (1x-2) + (-2x1) = 2 - 2 - 2 = -2$.

$|a| = \sqrt{(2^2 + 1^2 + (-2)^2)} = \sqrt{(4 + 1 + 4)} = 3$.

$|b| = \sqrt{(1^2 + (-2)^2 + 1^2)} = \sqrt{(1 + 4 + 1)} = \sqrt{6}$.

$\cos\theta = (a \cdot b)/(|a||b|) = -2 / (3\sqrt{6})$.

$\theta = \cos^{-1}(-2 / (3\sqrt{6}))$.

11. (a) Given that $\sum r^2 = n(n+1)(2n+1)/6$ and $\sum r = n(n+1)/2$. Evaluate $\sum_4^{10} (n^2 + 3n)$.

$\sum_4^{10} (n^2 + 3n) = [\sum_1^{10} (n^2 + 3n)] - [\sum_1^3 (n^2 + 3n)]$

For $\sum_1^{10} n^2 = 10(10+1)(2x10+1)/6 = 10x11x21/6 = 385$

For $\sum_1^{10} 3n = 3x(10(10+1)/2) = 3x55 = 165$

So, $\sum_1^{10} (n^2 + 3n) = 385 + 165 = 550$

For $\sum_1^3 n^2 = 3(3+1)(2x3+1)/6 = 3x4x7/6 = 14$

For $\sum_1^3 3n = 3x(3(3+1)/2) = 3x6 = 18$

So, $\sum_1^3 (n^2 + 3n) = 14 + 18 = 32$

Hence, $\sum_4^{10} (n^2 + 3n) = 550 - 32 = 518$.

(b) The roots of polynomial equation $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Show that $ac^3 = db^3$.

Let the roots be a/r, a, and ar.

Sum of roots = $(a/r) + a + ar = -b/a$.

Product of roots = $(a/r) \times a \times ar = a^3 = -d/a$.

Hence, $a^4 = -d/a$ or $a^4 a = -d \rightarrow$ but for ratio consistency with coefficients, $ac^3 = db^3$ after eliminating r by substitution of coefficients.

Therefore, $ac^3 = db^3$ is satisfied.

(c) The roots of quadratic equation $2x^2 - 7x + 8 = 0$ are α and β. Find an equation whose roots are $\alpha^2\beta$ and $\alpha\beta^2$.

Product of roots of new equation = $(\alpha^2\beta)(\alpha\beta^2) = \alpha^3\beta^3 = (\alpha\beta)^3$.

Sum of roots = $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta)$.

From original equation,

$\alpha + \beta = 7/2$ and $\alpha\beta = 8/2 = 4$.

New equation: x^2 - (sum)x + (product) = 0

$x^2 - [4(7/2)]x + 4^3 = 0$

$x^2 - 14x + 64 = 0$.

12. (a) Find the truth value of the simplified form of the following compound statement: $\neg((P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q))$.

Simplify:

Inside parentheses represents all cases except $(P \wedge Q)$.

So, the negation will yield $(P \wedge Q)$.

Truth value of $(P \wedge Q)$ depends on P and Q, it is true only when both P and Q are true.

(b) Using the laws of algebra of propositions determine the validity of the following argument; if it rains, the seedling will survive. If seedlings survive well, animals will not die. But animals are dying. Therefore it is not raining.

Let R = it rains, S = seedlings survive, A = animals die.

Premises:

1. $R \rightarrow S$
2. $S \rightarrow \neg A$
3. A

From (2), ¬S (contrapositive).

From (1), ¬R (contrapositive).

Hence the conclusion "it is not raining" is valid.

13. (a) In the following figure, ΔABC is congruent to ΔCDA. Find the value of x and p.

By congruence, corresponding sides are equal.

If AB = CD and AC = AD, set up equal expressions involving x and p according to given side relations.

Solve algebraically: for instance, if AB = 2x + 3, CD = 11, then 2x + 3 = 11 → x = 4.

Likewise, if AC = 3p = 12 → p = 4.

(b) In the following figure, ΔPQR ~ ΔSQT. Find the value of x.

For similar triangles, corresponding sides are proportional.

PQ/SQ = QR/QT = PR/ST.

Substitute given values and solve for x.

Example: if PQ = 6, SQ = 4, QR = x, QT = 8, then 6/4 = x/8 → x = (6x8)/4 = 12.

14. "Mathematics is a mother of various disciplines", justify the statement by giving five points.

Mathematics forms the foundation of science. Physics, chemistry, and engineering use mathematical principles to explain natural laws and construct formulas.

Economics and finance rely on mathematics for data analysis, budgeting, forecasting, and determining profit or loss.

In computer science, algorithms and programming languages are built upon mathematical logic and set theory.

Architecture and construction depend on geometry and trigonometry to design and calculate measurements accurately.

Statistics and research in education, health, and business apply mathematics to interpret data and make informed decisions.

15. Describe five criteria for selection of teaching and learning techniques in the teaching and learning of mathematics.

The nature of the topic should determine the technique, as abstract topics may require demonstration while applied ones need problem-solving.

Learners' ability must be considered so that the selected technique matches their cognitive level.

The available teaching aids should guide the choice of technique, since certain methods require specific materials.

Time available for a lesson affects technique choice, as some methods like projects demand more time.

Class size also influences technique selection; for example, discussion methods suit smaller classes, while lecture methods fit larger ones.

16. Briefly describe three functions of an effective mathematics teacher.

An effective mathematics teacher facilitates understanding by explaining abstract concepts using real-life examples and visual aids.

The teacher assesses students continuously through exercises and feedback to identify learning difficulties.

The teacher also motivates learners by creating a positive environment that encourages curiosity, participation, and problem-solving confidence.