THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATION COUNCIL

DIPLOMA IN SECONDARY EDUCATION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours. **SOLUTIONS** **Year: 2020**

Instructions

1. This paper consists of sections A, B and C.
2. Answer **all** questions from Section A and **two (2)** questions from each of section B and C.
3. Section A carries **40** marks, Section B and C carry 30 marks each.
4. Cellular phones are **not** allowed inside the examination room.
5. Write your **Examination Number** on every page of your answer booklet

1. List four disciplines where mathematics can be applied.

Mathematics can be applied in engineering. Engineers use mathematical concepts such as calculus, algebra, and geometry to design and analyze structures, machines, and systems.

Mathematics is applied in economics. Economists use mathematical models to study trends, forecast future developments, and analyze the relationships between different variables.

In computer science, mathematics is applied in algorithms, data analysis, artificial intelligence, and programming logic to ensure accuracy and efficiency in computations.

Mathematics is also applied in physics. It helps in describing physical phenomena, calculating forces, motion, and energy, and expressing natural laws through mathematical equations.

2. Using algebraic laws of propositions prove that $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

$(P \wedge Q) \Rightarrow (P \vee Q)$

$\equiv \neg(P \wedge Q) \vee (P \vee Q)$

$\equiv (\neg P \vee \neg Q) \vee (P \vee Q)$

$\equiv (\neg P \vee P) \vee (\neg Q \vee Q)$

$\equiv T \vee T$

$\equiv T$

Therefore, $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology.

3. Given that $a = 2i + 3j - k$ and $b = 4i + 6j + \lambda k$ are perpendicular.

(a) Find the value of λ.

$a \cdot b = 0$

$(2)(4) + (3)(6) + (-1)(\lambda) = 0$

$8 + 18 - \lambda = 0$

$\lambda = 26$

(b) Find the projection of vector a on vector b if $\lambda = 2$.

$a = 2i + 3j - k$

$b = 4i + 6j + 2k$

$a \cdot b = (2)(4) + (3)(6) + (-1)(2) = 8 + 18 - 2 = 24$

$|b| = \sqrt{(4^2 + 6^2 + 2^2)} = \sqrt{(16 + 36 + 4)} = \sqrt{56} = 2\sqrt{14}$

Projection = $(a \cdot b)/|b| = 24 / (2\sqrt{14}) = 12/\sqrt{14}$

4. Mention eight important parts of mathematics teachers' subject log book.

The teacher's name must be recorded to identify the owner of the log book.

The name of the school should appear to indicate the institution where the log book is used.

The class or stream taught must be written to specify the level of learners.

The subject title should be included to define the content covered.

The scheme of work section is important for planning weekly lessons.

The record of work covered helps in tracking the progress of topics taught.

The remarks section is included to note challenges or comments about lessons.

The supervisor's signature section is added to confirm that the log book has been reviewed.

5. Find the derivative of $y = (e^x \log a^x) / 7^x$.

$y = (e^x \log a^x) / 7^x$

$= x \log a \times (e^x / 7^x)$

$= (x \log a)(e^x)(7^{-x})$

dy/dx = log a x e^x x 7^(-x) + x log a x e^x x 7^(-x) x (1 - ln7)

$dy/dx = (e^x \log a)(7^{-x}) [1 + x(1 - \ln 7)]$

6. In the following figure prove that $\triangle XYZ$ is similar to $\triangle XMN$ and hence calculate the lengths MN and MY.

To prove similarity, corresponding angles of the triangles are equal, and their sides are proportional. Therefore, $\triangle XYZ \sim \triangle XMN$.

Using side ratios, MN/XY = XM/XX = XN/XZ.

If XY = 8 cm, XZ = 12 cm, and XM = 4 cm, then MN = (XM/XY) x YZ = (4/8) x YZ = YZ/2.

If YZ = 10 cm, then MN = 5 cm.

For MY, using the ratio of corresponding sides, MY/XY = XM/XX.
MY = (XM/XX) x XY = (4/8) x 8 = 4 cm.

7. (a) In how many ways can three students be selected out of group of ten students.

Number of selections = 10C3 = 10! / (3! x 7!) = (10 x 9 x 8) / (3 x 2 x 1) = 120

(b) How many different colours can be formed from mixing any two out of red, orange, yellow, green, and blue if no colour is repeated in any mixture.

Number of colour mixtures = 5C2 = 5! / (2! x 3!) = (5 x 4) / 2 = 10

8. With the aid of diagram how can you guide Form Three students to prove the chord property that "The perpendicular bisector of a chord passes through the center of a circle".

Draw a circle with center O and a chord AB. Draw the perpendicular bisector of AB that meets it at point M. Connect points O and M.
Since OA = OB (radii of the same circle), triangles OMA and OMB are congruent by the RHS criterion.
Hence, OM is perpendicular to AB and passes through the center of the circle, proving the chord property.

9. Evaluate the following series:

(a) ∑(n=1→∞) 1/[n(n+3)]

1/[n(n+3)] = 1/3 [1/n - 1/(n+3)]
Partial sum telescopes to 1/3 (1 + 1/2 + 1/3) = 11/18

(b) ∑(n=1→∞) (-1)^n x n^2

The terms alternate and increase in magnitude without bound. Hence, the series diverges.

10. Define the following terms as applied in Linear Programming:

A Linear Programming problem is a mathematical model used to maximize or minimize a linear objective function subject to linear constraints.

An Optimization problem is a situation where one seeks the best solution among many possible ones according to a certain criterion.

A Feasible region is the set of all points that satisfy the constraints in a linear programming problem.

An Optimal solution is the point within the feasible region that gives the best value of the objective function.