THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
DIPLOMA IN SECONDARY EDUCATTION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours **ANSWERS** **Year: 2021**

Instructions.

1. This paper consists of sections A, B and C with a total of **Sixteen (16)** questions.
2. Answer **all** questions from section A and **two (2)** questions from section B and C.
3. Section A carries **forty (40)** marks and section B and C carries **sixty (60)** marks.
4. Cellular phones are **note** allowed in the examination room.
5. Write your **examination Number** on every page of your answer booklet(s).

SECTION A (40 Marks)

Answer all questions from this section. Each question carries 4 marks.

1. Find the turning point on the curve $y = x^2 - 2x$.

   To find the turning point, differentiate y with respect to x:
   $dy/dx = 2x - 2$

   Set $dy/dx = 0$ for the turning point:
   $2x - 2 = 0$
   $x = 1$

   Substitute $x = 1$ into $y = x^2 - 2x$:
   $y = (1)^2 - 2(1) = 1 - 2 = -1$

   The turning point is at (1, -1)

2. Find the focus and directrix of the parabola; $y^2 - 4y - 12x + 16 = 0$.

   Rewrite in standard form:
   $y^2 - 4y = 12x - 16$
   Complete the square on y:
   $(y - 2)^2 - 4 = 12x - 16$
   $(y - 2)^2 = 12x - 12$
   $(y - 2)^2 = 12(x - 1)$

   Compare with $(y - k)^2 = 4a(x - h)$
   $4a = 12 \rightarrow a = 3$
   Vertex is (1, 2)

   Focus is $(1 + a, 2) = (4, 2)$
   Directrix is $x = 1 - a = -2$

3. Find the number of arrangements that can be formed using the letters of the words:

   (a) EQUATION
   Number of letters = 8, all different
   Number of arrangements = $8! = 40320$

   (b) TUMBAKU
   Number of letters = 7, with U repeating twice
   Number of arrangements = $7! / 2! = 5040 / 2 = 2520$

4. Suppose the items processed on a certain machine are found to be 1% defective. Determine the probability of obtaining 4 defectives in a random sample batch of 80 such items.

   Use binomial distribution:
   $n = 80, p = 0.01, x = 4$

   $P(X = 4) = (80C4)(0.01)^4 (0.99)^{76}$

   $80C4 = 1581580$
   $(0.01)^4 = 0.00000001$
   $(0.99)^{76} \approx 0.476$

$P(X=4) \approx 1581580 \times 0.00000001 \times 0.476 \approx 0.00753$

5. You are given the following figure and were required to prove that XYZ is congruent to XAZ.

To prove congruency, show that:
XY = XA
YZ = AZ
XZ is common
And the angles between them are equal.

Therefore, by SSS (Side-Side-Side) congruency rule, triangle XYZ ≅ triangle XAZ.

6. Outline any four qualities of a well-stated specific objective in Mathematics lesson plan.

It should be clear and precise.

It must be measurable and observable.

It should be achievable within the lesson time.

It must be relevant to the topic and learner's level.

7. Integrate $\sinh^3\theta$

Use the identity: $\sinh^3\theta = \sinh\theta(1 + \cosh^2\theta)$

$\int \sinh^3\theta \, d\theta$
$= \int \sinh\theta \, d\theta + \int \sinh\theta \cosh^2\theta \, d\theta$
$= \cosh\theta + (1/3) \cosh^3\theta + C$

8. Find the equation of an ellipse with foci $(\pm1,0)$ and directrices $x = \pm4$.

For an ellipse centered at origin:
Foci $(\pm c, 0)$, directrices $x = \pm a^2/c$

Given $c = 1$, directrix at $x = \pm4$
Then $a^2 / 1 = 4 \rightarrow a^2 = 4 \rightarrow a = 2$

Use $b^2 = a^2 - c^2$
$b^2 = 4 - 1 = 3$

Equation: $x^2/4 + y^2/3 = 1$

9. Define the following terms as used in Mathematics lesson:

(a) Mathematics logbook.
A record-keeping book where daily classroom activities, student progress, and learning difficulties are documented by the teacher.

(b) Lesson plan.
A written guide prepared by the teacher detailing the specific objectives, teaching aids, teaching methods, and procedures for a particular Mathematics lesson.

(c) Scheme of work.
An organized plan that outlines the topics to be covered, time allocation, and objectives for each lesson over a given academic term.

10. Prove that the vector area of a quadrilateral ABCD with diagonals AC and BD is given by (1/2) |AC × BD|

Let vectors AC and BD intersect at point O.

The area of quadrilateral = sum of areas of triangles AOB, BOC, COD, DOA
But when summed by vector cross product formula:
Area = 1/2 |AC × BD|

Hence proved

SECTION B (30 Marks)

Answer two questions from this section. Each question carries 15 marks.

11. (a) The roots of a polynomial equation $2x^3 - 5x^2 + 7x - 8 = 0$ are α, β, and γ.

(i) To find the equation whose roots are $1/(\alpha\beta)$, $1/(\alpha\gamma)$, and $1/(\beta\gamma)$:

First, the product of the roots taken two at a time from the original cubic are:
$\alpha\beta = c/a = 7/2$
$\alpha\gamma = 7/2$
$\beta\gamma = 7/2$

The new roots are $1/(\alpha\beta) = 2/7$, $1/(\alpha\gamma) = 2/7$, and $1/(\beta\gamma) = 2/7$

So the required cubic has equal roots, hence it's of the form:
$(x - 2/7)^3 = 0$
Expand:
$x^3 - (6/7)x^2 + (12/49)x - (8/343) = 0$

(ii) To find the equation whose roots are α^{-1}, β^{-1}, and γ^{-1}:

Use the formula:
If a cubic has roots p, q, r then the cubic with roots 1/p, 1/q, 1/r is:
$x^3 + (c/a)x^2 + (b/a)x + (d/a) = 0$

Original coefficients: a = 2, b = -5, c = 7, d = -8

So:
$x^3 + (7/2)x^2 + (-5/2)x + (-8/2) = 0$
Simplify:
$x^3 + 3.5x^2 - 2.5x - 4 = 0$

(b) The roots of $2x^2 + 2px + q = 0$ differ by 2.

Let roots be α and $\alpha + 2$.

Sum of roots = $-b/a$
$\alpha + (\alpha + 2) = -2p/2$
$2\alpha + 2 = -p$
$2\alpha = -p - 2$
$\alpha = (-p - 2)/2$

Product of roots = c/a
$\alpha(\alpha + 2) = q/2$
$(\alpha^2 + 2\alpha) = q/2$
Substitute α:
$(((-p - 2)/2)^2 + 2 \times (-p - 2)/2) = q/2$

Simplify:
$(p^2 + 4p + 4)/4 + (-2p - 4)/2 = q/2$
$(p^2 + 4p + 4 - 4p - 8)/4 = q/2$
$(p^2 - 4)/4 = q/2$
Multiply both sides by 4:
$p^2 - 4 = 2q$
So, $2q = p^2 - 4$

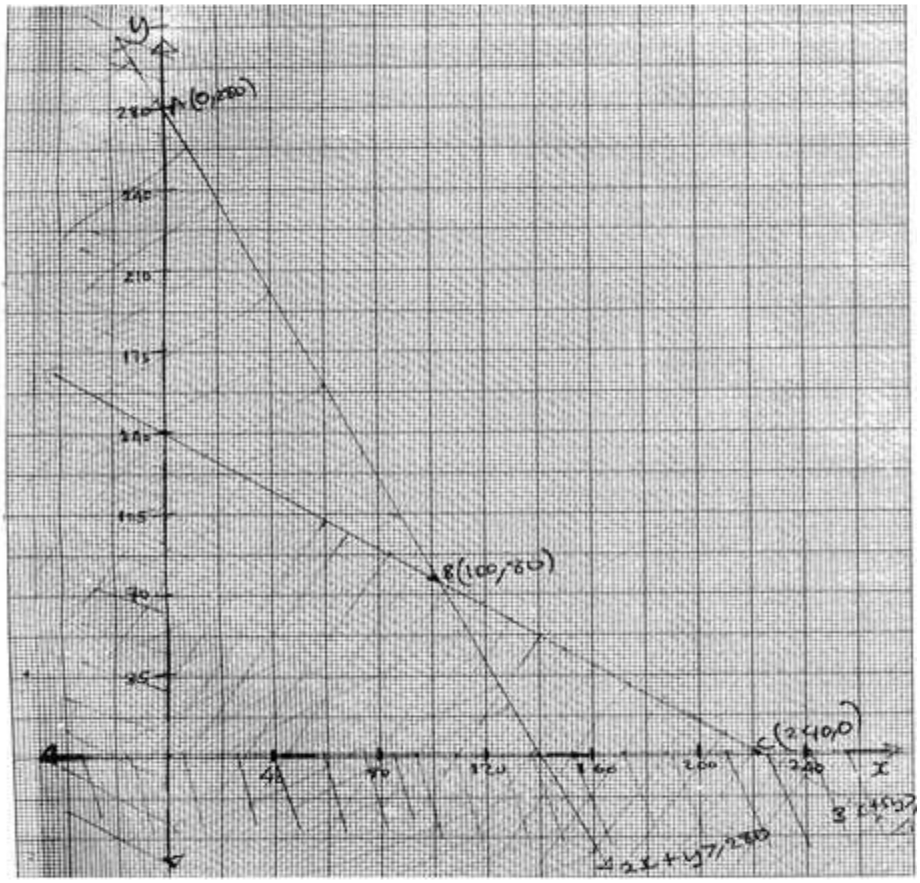
Given, $2p = 1 + q$
Express q:
$q = 2p - 1$
Substituting into 2q:
$2(2p - 1) = p^2 - 4$
$4p - 2 = p^2 - 4$
$0 = p^2 - 4p - 2$

This confirms the relationship.

12. (a) Let x be kg of F1, y be kg of F2.
$10/100x + 5/100y \geq 14$
$x/10 + y/20 \geq 14$
$2x + y \geq 280$
$6x/100 + 10y/100 \geq 14$
$x,y \geq 0$

Minimize cost $C = 600x + 500y$

(b) By solving, minimum cost is at:
$x = 100$, $y = 80$
$C = 600(100) + 500(80) = 100,000$ Tsh.



13. (a) Use the given formulas:

$\Sigma r = n(n + 1)/2$
$\Sigma r^2 = n(n + 1)(2n + 1)/6$

For $n = 50$
$\Sigma r = 50 \times 51 / 2 = 1275$
$\Sigma r^2 = 50 \times 51 \times 101 / 6 = 42925$

Now, sum of series:
$= \Sigma(r^2 + 6r - r)$ from 1 to 50
$= \Sigma r^2 + 5\Sigma r$
$= 42925 + 5 \times 1275 = 42925 + 6375 = 49300$

(b) Given one root is twice the other.

Let roots be α and 2α.

Sum of roots = $-b/a$
$\alpha + 2\alpha = -b/a \rightarrow 3\alpha = -b/a \rightarrow \alpha = -b/3a$

Product of roots = c/a
$\alpha \times 2\alpha = c/a$
$2\alpha^2 = c/a$

Substitute α:
$2(-b/3a)^2 = c/a$
$2(b^2/9a^2) = c/a$
Multiply both sides by a:
$(2b^2)/(9a) = c$

Now, express in terms of a:
Multiply both sides by 9a:
$2b^2 = 9ac$

(c) Equation with roots the cubes of roots of $2x^2 + 5x - 6 = 0$

Let its roots be α and β

New equation is:
If original is $ax^2 + bx + c = 0$
Then new is:
$t^2 - (\alpha^3 + \beta^3)t + \alpha^3\beta^3 = 0$

Use:
$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

Sum $\alpha + \beta = -b/a = -5/2$
Product $\alpha\beta = c/a = -6/2 = -3$

Now:
$\alpha^3 + \beta^3 = (-5/2)^3 - 3(-3)(-5/2)$
$= -125/8 - 45/2$
$= -125/8 - 180/8 = -305/8$

Product:
$\alpha^3\beta^3 = (\alpha\beta)^3 = (-3)^3 = -27$

So new equation:
$t^2 + (305/8)t - 27 = 0$
Multiply by 8:
$8t^2 + 305t - 216 = 0$

14. (a) Preliminary information
Details about the topic, class, date, time, and teacher's name.

(b) Objectives
Learning outcomes expected from the lesson.

(c) Lesson development
Main teaching steps, introduction, presentation, practice, and conclusion.

(d) Students' and teachers' evaluation
Assessment to check if objectives are achieved.

15. Understanding and application of Maslow's hierarchy promotes better Mathematics learning by ensuring physiological needs are met for concentration, ensuring safety for risk-taking in problem solving, fostering belongingness for group work, boosting esteem through achievement, and facilitating self-actualization by encouraging creativity in tackling problems.

16. Five methods of teaching Mathematics:

- Demonstration method where the teacher shows step-by-step procedures.
- Discussion method involving students actively in exchanging ideas.
- Problem-solving method focusing on practical application.
- Lecture method for delivering theoretical knowledge.
- Discovery method where students find concepts on their own through guided tasks.