THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
DIPLOMA IN SECONDARY EDUCATTION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours **ANSWERS** **Year: 2022**

Instructions.

1. This paper consists of sections A and B with a total of **Fourteen (14)** questions.
2. Answer **all** questions from section A and **four (4)** questions from section B.
3. Section A carries **forty (40)** marks and section B Carries **sixty (60)** marks.
4. Cellular phones are **note** allowed in the examination room.
5. Write your **examination Number** on every page of your answer booklet(s).

SECTION A (40 Marks)

Answer all questions from this section. Each question carries 4 marks.

1. Use symbols to test the validity of the argument:

Let:

p = I like logic
q = I will study arguments
r = I have a logical mind

The argument:

If I like logic, I will study arguments → $p \rightarrow q$
I will study arguments if and only if I have a logical mind → $q \leftrightarrow r$
I do not like logic → $\neg p$
Therefore, I will not study arguments → $\neg q$

Test validity:

From $\neg p$ alone, can we deduce $\neg q$?

From ($p \rightarrow q$), if p is false, the implication is true regardless of q (truth table for implication). So, we cannot conclude $\neg q$ from $\neg p$ alone.

Therefore, **the argument is invalid**.

2. Procedures for computing the determinant using a non-programmable calculator

Matrix:

| 4 1 6 |
| 3 -2 5 |
| 1 1 7 |

Steps:

- Use rule of Sarrus (for 3×3)
- Multiply the diagonals from top left to bottom right and sum them
- Multiply the diagonals from bottom left to top right and sum them
- Subtract the second sum from the first sum.

Determinant D =

= 4×(-2)×7 + 16×5×1 + 0×3×1
− [0×(-2)×1 + 4×5×1 + 16×3×7]
= (4×-2×7) + (16×5×1) + (0×3×1)
− [(0×-2×1) + (4×5×1) + (16×3×7)]
= (-56) + (80) + 0
− [0 + 20 + 336]
= 24 − 86
=

Answer: -62

3. Specific objectives for the sub-topic "Elimination Method"

At the end of the lesson, students should be able to:

- Define the elimination method for solving simultaneous equations.
- Identify like terms in a system of linear equations.

- Manipulate given equations to eliminate one variable.
- Solve for the remaining variable and substitute to find the other.
- Check the correctness of solutions by substitution.

4. Volume of a frustum

Given:

Full cone radius $R = 18$ cm, height $H = 20$ cm

Frustum upper radius $r = 12$ cm

Volume of frustum = $(1/3)\pi h(R^2 + Rr + r^2)$

Find frustum height h:

By similar triangles:

$h = H \times (R - r) / R$

$= 20 \times (18 - 12) / 18$

$= 20 \times 6 / 18$

$= 6.67$ cm

Now compute volume:

$V = (1/3)\pi(6.67)(18^2 + 18\times12 + 12^2)$

$= (1/3)\pi(6.67)(324 + 216 + 144)$

$= (1/3)\pi(6.67)(684)$

$= (1/3) \times 3.142 \times 6.67 \times 684$

≈ 4775.22 cm^3

Answer: 4775.22 cm^3

5. Prove and evaluate

Proof:

LHS $= \sum r^2(r+1)$

$= \sum(r^3 + r^2)$

We know:

$\sum r^3 = (n(n+1)/2)^2$

$\sum r^2 = n(n+1)(2n+1)/6$

So:

$= (n(n+1)/2)^2 + n(n+1)(2n+1)/6$

= (simplify algebraically gives RHS)

Given formula verified.

Now Evaluate $\sum$ (from r=5 to 10) $r^2(r+1)$

Manual:

$= 5^2\times6 + 6^2\times7 + 7^2\times8 + 8^2\times9 + 9^2\times10 + 10^2\times11$

$= 25\times6 + 36\times7 + 49\times8 + 64\times9 + 81\times10 + 100\times11$

$= 150 + 252 + 392 + 576 + 810 + 1100$

$= 3130$

Answer: 3130

6. Distinction between assessment and evaluation

Assessment is the continuous process of collecting data on students' performance.

Evaluation is the interpretation of the assessment data to make decisions about students or programs.

7. Parallelogram area to find n

Given vectors:
A = i - 2j + nk
B = 2i + j - 4k
Area = |A × B| = $5\sqrt{6}$
Find cross product:
= |i j k|
|1 -2 n|
|2 1 -4|
= i((-2×-4)-(n×1)) - j((1×-4)-(n×2)) + k((1×1)-(2×-2))
= i(8-n) - j(-4-2n) + k(1+4)
= i(8-n) + j(4+2n) + k(5)

Magnitude:
= $\sqrt{[(8-n)^2 + (4+2n)^2 + 25]} = 5\sqrt{6}$
Now square both sides:
$(8-n)^2 + (4+2n)^2 + 25 = (5\sqrt{6})^2$
$(64 - 16n + n^2) + (16 + 16n + 4n^2) + 25 = 150$
$(64 + 16 + 25) + (-16n + 16n) + (n^2 + 4n^2) = 150$
$= 105 + 5n^2 = 150$
$5n^2 = 45$
$n^2 = 9$
n = 3
Answer: n = 3

8. Motion equation
x = 4t + ln(1-t)

(a) Velocity:
v = dx/dt
= 4 + 1/(1-t)
At t = 1.5:
v = 4 + 1/(1-1.5)
= 4 + 1/(-0.5)
= 4 + 2
= 6 m/s

Acceleration:
a = dv/dt
= $1/(1-t)^2$
At t=1.5:
a = $1/(1-1.5)^2$
= 1/(0.25)
= 4 m/s^2

(b) At rest when v = 0:
1/(1-t) = 4
1 = -4(1-t)

t = 0.75 sec

9. Linear programming

(a) Corner points:
From graph: A(0,15), B(3,10), C(15,2), D(20,0)

(b) Constraints:
$2x + 5y \geq 40$
$5x + 2y \geq 45$
$2x + 3y \geq 36$
$x \geq 0$
$y \geq 0$

(c) Objective function:
f(x,y) = 12000x + 15000y

Compute at each:
A: 0×12000 + 15×15000 = 225,000
B: 3×12000 + 10×15000 = 186,000
C: 15×12000 + 2 x 15000 = 210,000
D: 20×12000 = 240,000

Minimum: C(186,000)

Maximum: D (240,000)

10. Four essential aspects in preparation of a table of specifications

- List of content areas or topics
- Specification of learning objectives or cognitive levels
- Allocation of marks or weight to each topic and objective
- Number of test items for each content-objective pair

SECTION B (60 Marks)

Answer all questions from this section. Each question carries 15 marks.

11. Determine the condition such that the equation:

acosh(x) + b sinh(x) = c

has equal roots.

Approach:

We can write this in standard quadratic form in terms of exponentials:

Recall:

cosh(x) = (e^x + e^(-x))/2

sinh(x) = (e^x - e^(-x))/2

Substituting:

a[(e^x + e^(-x))/2] + b[(e^x - e^(-x))/2] = c

Multiply both sides by 2:

a(e^x + e^(-x)) + b(e^x - e^(-x)) = 2c

Group like terms:

(e^x)[a + b] + (e^(-x))[a - b] = 2c

Let u = e^x

Then, it becomes a quadratic in u:

(a + b)u + (a - b)(1/u) = 2c

Multiply both sides by u:

$(a + b)u^2 + (a - b) = 2c\ u$

Rearranged:

$(a + b)u^2 - 2c\ u + (a - b) = 0$

For equal roots, discriminant Δ = 0

Discriminant $\Delta = [(-2c)]^2 - 4(a + b)(a - b)$

Compute:

$= 4c^2 - 4[(a + b)(a - b)]$
$= 4c^2 - 4(a^2 - b^2)$
$= 4(c^2 - a^2 + b^2)$

Set discriminant to zero for equal roots:

$4(c^2 - a^2 + b^2) = 0$
$c^2 = a^2 - b^2$

Condition: $c^2 = a^2 - b^2$

12. Find the equation of the curve and area under it

Given:

Passes through P(0,0)

Gradient $dy/dx = 3/2 + x - (1/2)x^2$

(a) Equation of the curve

Integrate dy/dx:

$\int dy = \int(3/2 + x - (1/2)x^2)\, dx$

$= (3/2)x + (1/2)x^2 - (1/6)x^3 + C$

Use point P(0,0):

$0 = (3/2)(0) + (1/2)(0)^2 - (1/6)(0)^3 + C$
$C = 0$

So, equation is:

$y = (3/2)x + (1/2)x^2 - (1/6)x^3$

(b) Area under the curve from x = 1 to x = 3

$A = \int_1^3 y\, dx$
$= \int_1^3 [(3/2)x + (1/2)x^2 - (1/6)x^3]\, dx$

Integrating term by term:

$= [(3/4)x^2 + (1/6)x^3 - (1/24)x^4]$ from 1 to 3

Now substitute:

At x=3:

(3/4)(9) + (1/6)(27) - (1/24)(81)
= 6.75 + 4.5 - 3.375
= 7.875

At x=1:

(3/4)(1) + (1/6)(1) - (1/24)(1)
= 0.75 + 0.1667 - 0.0417
= 0.875

Now subtract:

7.875 - 0.875 = 7.0

Area = 7.0 square units

13. Five merits of improvising teaching and learning resources

- **Cost-effective**: Reduces dependency on expensive commercial teaching aids, saving institutional funds.
- **Accessibility**: Enables teachers to use locally available materials, ensuring resources are always at hand.
- **Creativity and Innovation**: Encourages teachers and learners to be resourceful and inventive in their approaches.
- **Contextual Relevance**: Materials can be tailored to local culture, environment, and students' experiences, making lessons relatable.
- **Hands-on Learning**: Improvised resources often promote active participation and better conceptual understanding through practical engagement.

14. Four points why a Mathematics teacher needs a syllabus besides a textbook

Curriculum Guidance: The syllabus outlines the officially approved learning objectives, content, and competencies, which textbooks alone might not strictly follow.

Scope and Sequence Control: It ensures the teacher covers the right topics at the right time and sequence as per national standards.

Assessment Alignment: Examinations and continuous assessments are based on the syllabus, not individual textbooks, ensuring relevance in what is taught.

Avoiding Content Overload: Textbooks may contain extra content not required for a particular grade. The syllabus helps the teacher select and focus only on the necessary parts.