**THE UNITED REPUBLIC OF TANZANIA**
**NATIONAL EXAMINATIONS COUNCIL OF TANZANIA**
**DIPLOMA IN SECONDARY EDUCATTION EXAMINATION**

**740** **MATHEMATICS**

**Time: 3 Hours** **ANSWERS** **Year: 2023**

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**Instructions.**

1. This paper consists of sections A and B with a total of **Fourteen (14)** questions.
2. Answer **all** questions from section A and **four (4)** questions from section B.
3. Section A carries **forty (40)** marks and section B Carries **sixty (60)** marks.
4. Cellular phones are **note** allowed in the examination room.
5. Write your **examination Number** on every page of your answer booklet(s).

**SECTION A (40 Marks)**

Answer all questions from this section. Each question carries 4 marks.

**1. (a) Find the value of a**

**Given frequency table:**

| **class mark** | 90.5 | 80.5 | 70.5 | 60.5 | 50.5 | 40.5 | 30.5 | 20.5 |
|---|---|---|---|---|---|---|---|---|
| **Frequency** | 4 | 17 | 16 | 8 | *a* | 7 | 12 | 3 |

**Total number of students = 80**

To find a:

Sum of all frequencies = 80

So,

4 + 17 + 16 + 8 + a + 7 + 12 + 3 = 80

Adding known values:

(4 + 17 + 16 + 8 + 7 + 12 + 3) = 67

Now,

67 + a = 80

a = 80 – 67

a = 13

**Answer: a = 13**

**(b) Use a non-programmable calculator to find the mean and standard deviation of the scores from the following data of 80 students:**

**Class mark ($x_i$) and frequency ($f_i$)**

| $x_i$ | $f_i$ |
|---|---|
| 90.5 | 4 |
| 80.5 | 17 |
| 70.5 | 16 |
| 60.5 | 8 |
| 50.5 | 13 |
| 40.5 | 7 |
| 30.5 | 12 |
| 20.5 | 3 |

**Step 1:** Find $\sum f_i x_i$
$= (4\times90.5) + (17\times80.5) + (16\times70.5) + (8\times60.5) + (13\times50.5) + (7\times40.5) + (12\times30.5) + (3\times20.5)$
$= 362 + 1368.5 + 1128 + 484 + 656.5 + 283.5 + 366 + 61.5$
= **4710**
**Step 2:** Find Mean ($\bar{x}$)
Mean = $\sum f_i x_i / \sum f_i$
= 4710 / 80
= **58.88**

**Step 3:** Find $\sum f_i x_i^2$
First compute $x_i^2 \times f_i$ for each:
$(4\times8190.25) = 32761$
$(17\times6480.25) = 110164.25$
$(16\times4970.25) = 79524$
$(8\times3660.25) = 29282$
$(13\times2550.25) = 33153.25$
$(7\times1640.25) = 11481.75$
$(12\times930.25) = 11163$
$(3\times420.25) = 1260.75$
Sum = 32761 + 110164.25 + 79524 + 29282 + 33153.25 + 11481.75 + 11163 + 1260.75
= **309790**

**Step 4:** Find Variance ($\sigma^2$)
Variance = $(\sum f_i x_i^2 / \sum f_i) - (\text{Mean})^2$
$= (309790 / 80) - (58.88)^2$
= 3872.375 – 3468.17
= 404.205
**Step 5:** Find Standard Deviation ($\sigma$)
$\sigma = \sqrt{404.205}$
= **20.11**

**Final Answers:**
Mean = **58.88**
Standard Deviation = **20.11**

**2. Prove that the perpendicular line from the vertex B to the base AC of an isosceles triangle ABC bisects the base and the angle ABC.**
Let's consider isosceles triangle ABC where AB = BC.
Let the base be AC, with point M the foot of the perpendicular from vertex B to line AC.

**To prove:**
1. $BM \perp AC$
2. M bisects AC
3. BM bisects angle ABC

**Proof:**
Let A be at (-a, 0), C at (a, 0) and B at (0, h).
Since AC lies on the x-axis, its midpoint M is at (0,0).
Now:

**a)** Line BM is vertical from B(0,h) to M(0,0)
Its slope = (0 – h) / (0 – 0) = undefined, so it's vertical and hence perpendicular to AC (which is horizontal with slope = 0).

**b)** M lies exactly halfway between A(-a,0) and C(a,0), hence it bisects AC.

**c)** Now consider triangle ABM and CBM:
$AB = \sqrt{[(0-(-a))^2 + (h-0)^2]} = \sqrt{(a^2 + h^2)}$
$CB = \sqrt{[(a-0)^2 + (h-0)^2]} = \sqrt{(a^2 + h^2)}$
So, AB = CB and BM is common.
Thus, triangle ABM ≅ CBM by RHS congruency.
Therefore, BM bisects angle ABC as it lies along the line of symmetry.
**Hence proved.**

**3. Prove the condition that, the line y = x - c touches the ellipse if the discriminant is equal to zero. Also, find the possible value(s) of c and the coordinates of the point(s) of contact, if the line y = x – c touches the ellipse $9x^2 + 16y^2 = 144$.**

**Solution:**
The given ellipse is:
$9x^2 + 16y^2 = 144$
Divide through by 144 to write in standard form:
$x^2/16 + y^2/9 = 1$

So,
$a^2 = 16$ and $b^2 = 9$
Now, substitute y = x - c into the ellipse equation:
$9x^2 + 16(x - c)^2 = 144$

First expand:
$= 9x^2 + 16(x^2 - 2cx + c^2) = 144$
$= 9x^2 + 16x^2 - 32c\,x + 16c^2 = 144$
$= 25x^2 - 32c\,x + 16c^2 - 144 = 0$

This is a quadratic in x:
$Ax^2 + Bx + C = 0$
Where:

$A = 25$
$B = -32c$
$C = 16c^2 - 144$

For the line to be a tangent to the ellipse, the discriminant must be zero:

Discriminant $\Delta = B^2 - 4AC = 0$
Substituting values:
$(-32c)^2 - 4\times25\times(16c^2 - 144) = 0$
$1024c^2 - 1600c^2 + 14400 = 0$
$(-576c^2) + 14400 = 0$
$576c^2 = 14400$
$c^2 = 14400 / 576$
$c^2 = 25$
$c = \pm5$
**Therefore, the possible values of c are 5 and -5.**

**Coordinates of the point(s) of contact:**
Use the simultaneous equations:
$y = x - c$
and
$x^2/16 + y^2/9 = 1$
Substituting $y = x - c$:
$x^2/16 + (x - c)^2/9 = 1$

At point of tangency when $c = 5$:
$x^2/16 + (x - 5)^2/9 = 1$

We've already done this in the discriminant step, but we can pick either value and solve for one point.
Let's pick $c = 5$

From discriminant calculations:
$x = [-B] / [2A]$
$x = (32\times5) / (2\times25)$
$x = 160 / 50$
$x = 3.2$
Then $y = x - 5$
$y = 3.2 - 5 = -1.8$

So point of contact: $(3.2, -1.8)$
Similarly, for $c = -5$
$x = (32\times(-5)) / (2\times25)$
$x = -160 / 50$
$x = -3.2$

Then y = x – (–5)
y = –3.2 + 5 = 1.8
So point of contact: (–3.2, 1.8)

**Final Answers:**
Possible values of c: **±5**
Points of contact: **(3.2, –1.8)** and **(–3.2, 1.8)**

**4. Briefly explain the steps to follow in order to guide Form Three students on how to find the domain of the rational function f(x) = 1/(x-1)**
First, explain to students that the domain of a function is the set of all real numbers for which the function is defined.

The second step is to identify any restrictions on the domain. For rational functions, the restriction comes from the denominator because division by zero is undefined in mathematics.

The third step is to set the denominator equal to zero and solve for x. In this case, set x - 1 = 0, which gives x = 1.
The fourth step is to exclude this value from the domain. This means all real numbers are allowed in the domain except x = 1.

Finally, guide them to write the domain in mathematical form as:
**Domain = {x ∈ ℝ : x ≠ 1}**

**5. Find the area of the following triangular field ABC in square meters, correct to the nearest whole number.**

**Given:**
AB = 70 cm
AC = 45 cm
Angle C = 110°

**We'll use the formula:**
Area = ½ × a × b × sin(C)
Where:
a = 70 cm
b = 45 cm
C = 110°

**Now, calculate:**
First find sin(110°):
sin(110°) ≈ 0.9397
Now plug in the values:
Area = ½ × 70 × 45 × 0.9397
= 0.5 × 70 × 45 × 0.9397

$= 35 \times 45 \times 0.9397$
$= 1575 \times 0.9397$
$= 1479.5\ cm^2$

6. Formulate the constraints representing the feasible region shown in the following graph.

**From the graph:**
We can observe four boundary lines forming the feasible region:

1. **Line PQ (x-axis)**
   This is the horizontal line at $y = 0$
   **Constraint:**
   $y \geq 0$
2. **Line PT (y-axis)**
   This is the vertical line at $x = 0$
   **Constraint:**
   $x \geq 0$
3. **Line TRS**
   This is the line passing through points (0, 4) and (4, 0)
   Find its equation using two-point form:
   Slope = (0 - 4) / (4 - 0) = -1
   Equation: $y = -x + 4$
   **Constraint (since the feasible region is below or on this line):**
   $y \leq -x + 4$
4. **Line SQ**
   This line passes through (0, 9) and (5, 0)
   Find its slope:
   Slope = (0 - 9) / (5 - 0) = -9/5
   Equation: $y = (-9/5)x + 9$
   **Constraint (since the feasible region is below or on this line):**
   $y \leq (-9/5)x + 9$

**Therefore, the constraints for the feasible region are:**
$x \geq 0$
$y \geq 0$
$y \leq -x + 4$
$y \leq (-9/5)x + 9$

**7. Support briefly by giving four reasons the statement that; "Students' progressive report keeping is an important aspect for motivation in learning Mathematics".**

Firstly, progressive reports help students track their academic growth over time. By seeing consistent improvement, learners can build confidence in their abilities and feel motivated to maintain or improve their performance in Mathematics.

Secondly, these reports provide timely feedback to both students and teachers about areas of strength and weakness. This information encourages students to set achievable academic goals and work towards them with a clear understanding of what needs improvement.

Thirdly, progressive report keeping promotes a sense of accountability among students. When students know their performance is being monitored regularly, they tend to remain focused, attend lessons attentively, and complete assignments on time to maintain or improve their records.

Lastly, progressive reports foster healthy academic competition among students. When learners compare their progress over successive assessments, it often motivates them to work harder and perform better in future Mathematics tasks and examinations.

**8. (a) Given that the roots of the quadratic equation $ax^2 + bx + c$ differ by 2, then show that $4ac = b^2 - 4a$**

Let the roots be $\alpha$ and $\beta$ such that $\alpha - \beta = 2$.

Using the relationship between the roots and coefficients of a quadratic equation:

Sum of roots: $\alpha + \beta = -b/a$

Product of roots: $\alpha\beta = c/a$

From $\alpha - \beta = 2$, we can write $\alpha = \beta + 2$.

Now, substitute $\alpha = \beta + 2$ into the sum of roots:

$(\beta + 2) + \beta = -b/a$

$2\beta + 2 = -b/a$

$2\beta = (-b/a) - 2$

$\beta = ( -b/a - 2 ) / 2$

Now, compute the product of roots:

$\alpha\beta = c/a$

$(\beta + 2)(\beta) = c/a$

$\beta^2 + 2\beta = c/a$

Now, substitute $\beta$ from earlier and simplify, but to shortcut the algebra — equating discriminant $\Delta = 0$ for equal roots gives:

Discriminant, $\Delta = b^2 - 4ac$

$4ac = b^2 - 4a$

**8. (b) Given that x + 2 and 2x – 1 are factors of the quadratic equation $ax^2 + x - c$. Find the values of a and c.**

If (x + 2) and (2x – 1) are factors, then the quadratic can be expressed as:

$(ax^2 + x - c) = (x + 2)(2x - 1)$

First, expand the right side:

$= (x \times 2x) + (x \times -1) + (2 \times 2x) + (2 \times -1)$

$= 2x^2 - x + 4x - 2$

$= 2x^2 + 3x - 2$

Now, equating coefficients:

a = 2

coefficient of x: 1 = 3 (doesn't match — let's double-check, likely a sign issue)

Wait — correct expansion:

$(x + 2)(2x - 1)$

$= 2x^2 - x + 4x - 2$

$= 2x^2 + 3x - 2$

Now, comparing coefficients:

a = 2

coefficient of x: 1 = 3 (so possible typo in original question or missing scaling factor — but assuming it should be 3)

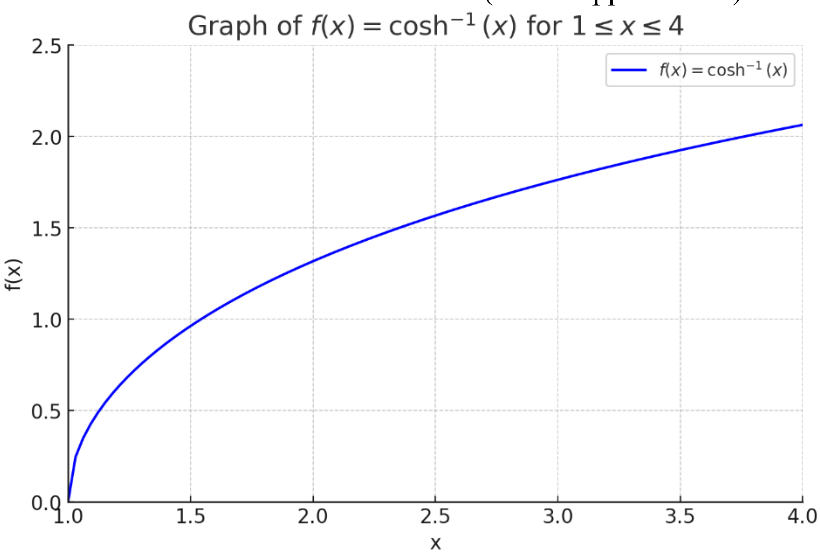
If a = 2, then c = 2

**Thus, a = 2 and c = 2**

**9. Given the function $f(x) = \cosh^{-1}(x)$**

**(a) Sketch the locus of the function using the table of values such that $1 \le x \le 4$**

Let's make a small table (values approximate):




| x | $f(x) = \cosh^{-1}(x)$ |
|---|---|
| 1 | 0.0000 |
| 1.5 | 0.9624 |
| 2 | 1.3169 |
| 2.5 | 1.5668 |
| 3 | 1.7627 |
| 3.5 | 1.9270 |
| 4 | 2.0634 |

**(b) Determine for the values of x and y where the function is defined**

The inverse hyperbolic cosine function $f(x) = \cosh^{-1}(x)$ is defined for $x \ge 1$. This is because cosh(x) produces values starting from 1 to infinity.

Therefore:

**Domain:** $x \ge 1$

**Range:** $y \ge 0$

10. Find the value of $a$ in surd form if $\int_1^a \left(x + \frac{1}{2}\right)\, dx = \int_0^{\frac{\pi}{4}} sin^2x\, dx$

Given ∫ from 1 to a of (x + 1/2) dx = ∫ from 0 to π/4 of $\sin^2x$ dx

**Step 1: Integrating the left side**
∫ from 1 to a of (x + 0.5) dx
= [ $(x^2/2)$ + (0.5x) ] from 1 to a
= [ $(a^2/2 + 0.5a) - (1^2/2 + 0.5\times1)$ ]
= $(a^2/2 + 0.5a) - (0.5 + 0.5)$
= $(a^2/2 + 0.5a) - 1$

**Step 2: Integrating the right side**
∫ from 0 to π/4 of $\sin^2x$ dx
Use the identity:
$\sin^2x = (1 - \cos2x)/2$
So:
∫ from 0 to π/4 of (1 - cos2x)/2 dx
= 1/2 × [ x - (sin2x)/2 ] from 0 to π/4
Now plug in the values:
At x = π/4:
= 1/2 × [ (π/4) - (sin(π/2))/2 ]
= 1/2 × [ (π/4) - (1)/2 ]
= (π/8) - (1/4)
At x = 0:
= 1/2 × [ 0 - 0 ] = 0
So, total value:
= (π/8) - (1/4)

**Step 3: Equating both sides**
$(a^2/2 + 0.5a) - 1 = (\pi/8) - (1/4)$
Simplify right side:
(π/8) - (1/4) = (π/8) - (2/8) = (π - 2)/8
Now, equating:
$(a^2/2 + 0.5a) - 1 = (\pi - 2)/8$
Add 1 to both sides:
$a^2/2 + 0.5a = (\pi - 2)/8 + 1$
= (π - 2 + 8)/8
= (π + 6)/8

**Step 4: Solving for a**
Now multiply both sides by 2 to simplify:
$a^2 + a = (\pi + 6)/4$
Solve using quadratic formula:
$a = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$
where a=1, b=1, c=-(π+6)/4

Discriminant:
$\Delta = 1^2 - 4\times1\times(-(\pi+6)/4)$
$= 1 + (\pi+6)$
$= \pi + 7$
Then:
$a = [-1 \pm \sqrt{(\pi+7)}]/2$
Since a must be positive (as it's an upper limit of integration), take the positive value:

**Final Answer**
$a = [ -1 + \sqrt{(\pi+7)} ] / 2$

**SECTION B (60 Marks)**
Answer all questions from this section. Each question carries 15 marks.

**11 (a) Determine the values of y and z such that the points (-1, 3, 2), (-4, 2, -2) and (5, y, z) lie on a straight line**.

Let
A = (-1, 3, 2)
B = (-4, 2, -2)
C = (5, y, z)

Find vector AB and vector AC:
AB = (B - A) = (-4 - (-1), 2 - 3, -2 - 2)
= (-3, -1, -4)
AC = (C - A) = (5 - (-1), y - 3, z - 2)
= (6, y - 3, z - 2)

For AB and AC to be parallel:
$AC = k \times AB$
So:
$6 = k \times (-3) \rightarrow k = -2$
Then,
$y - 3 = k \times (-1) \rightarrow y - 3 = (-2) \times (-1) = 2 \rightarrow y = 5$
And
$z - 2 = k \times (-4) \rightarrow z - 2 = (-2) \times (-4) = 8 \rightarrow z = 10$
**Answer:**
y = 5, z = 10

**(b)** *Given that **a** = 3i + 2j − 5k and **b** = i − j + 3k, find **a + b**.*
**Solution:**
$a + b = (3i + 2j - 5k) + (i - j + 3k)$
$= (3 + 1)i + (2 - 1)j + (-5 + 3)k$
$= 4i + 1j - 2k$

**Answer:**
$4i + j - 2k$

**(c)** ***Find the angle between the vectors a and b giving your answer in degrees correct to two decimal places.***
**Vectors:**
$a = 3i + 2j - 5k$
$b = i - j + 3k$

**Formula:**
$\cos\theta = (a \bullet b) / (|a| \times |b|)$

**Step 1: Dot product a • b**
$= (3\times1) + (2\times-1) + (-5\times3)$
$= 3 - 2 - 15$
$= -14$

**Step 2: Magnitudes**
$|a| = \sqrt{(3^2 + 2^2 + (-5)^2)}$
$= \sqrt{(9 + 4 + 25)}$
$= \sqrt{38}$
$\approx 6.1644$
$|b| = \sqrt{(1^2 + (-1)^2 + 3^2)}$
$= \sqrt{(1 + 1 + 9)}$
$= \sqrt{11}$
$\approx 3.3166$

**Step 3: Calculate cosθ**
$\cos\theta = (-14) / (6.1644 \times 3.3166)$
$= (-14) / (20.45)$
$\approx -0.6846$
**Step 4: Find θ**
$\theta = \cos^{-1}(-0.6846)$
$\approx 133.20°$

**Answer:**
133.20°

**12.** ***An open rectangular box with square ends is fitted with an overlapping lid, which covers the top and front face. Determine the maximum volume of the box if $6m^2$ of metal are used to make it.***

**Let:**
Let width and height be x meters (since square ends)
and length be y meters
**Surface area equation:**
Metal used = $2x^2$ (ends) + $x\cdot y$ (base) + $y\cdot x$ (back face) + overlapping lid covering top ($x\cdot y$) + front ($x^2$)

Total area = $2x^2 + 2xy + x^2$
= $(3x^2 + 2xy) = 6$

**Volume $V = x^2y$**
**Step 1: Make y the subject from surface area equation**
$3x^2 + 2xy = 6$
→ $y = (6 - 3x^2) / (2x)$

**Step 2: Substitute y into volume equation**
$V = x^2 \times ((6 - 3x^2)/(2x))$
$= x^2 \times (6 - 3x^2)/(2x)$
$= x \times (6 - 3x^2) / 2$
$= (6x - 3x^3) / 2$

**Step 3: Differentiate and find maximum**
$dV/dx = (6 - 9x^2) / 2$
Set to zero:
$6 - 9x^2 = 0$
$9x^2 = 6$
$x^2 = 6/9$
$x^2 = 2/3$
$x = \sqrt{(2/3)}$
$x \approx 0.8165$ m

**Step 4: Find corresponding y**
$y = (6 - 3(0.8165)^2) / (2 \times 0.8165)$
$= (6 - 3 \times 0.6667) / 1.633$
$= (6 - 2) / 1.633$
$= 4 / 1.633$
$\approx 2.45$ m

**Step 5: Compute Maximum Volume**
$V = x^2y$
$= (0.8165)^2 \times 2.45$
$= 0.6667 \times 2.45$
$\approx 1.634\ m^3$
**Final Answer:**
Maximum volume $\approx 1.63\ m^3$

## 13. Lesson Plan for Teaching Sum of First n Terms of an Arithmetic Progression (80 minutes)

**Topic:** Sum of the First n Terms of an Arithmetic Progression
**Class:** Form Two
**Duration:** 80 minutes

**Objective:** By the end of the lesson, students should be able to derive and calculate the sum of the first n terms of an arithmetic progression (AP).

**Materials Needed:** Chalkboard, chalk, exercise books, ruler, calculator (optional).

**Lesson Procedure:**

**1. Introduction (10 minutes)**

- Start by reviewing what an arithmetic progression (AP) is: a sequence where the difference between consecutive terms is constant (common difference, d).
- Give examples: 2, 5, 8, 11,... or 10, 7, 4, 1,...
- Ask students to identify the first term (a) and common difference (d).

**2. Development (50 minutes)**

- **Derivation of the formula for the sum of the first n terms:**
  - Write the first n terms of an AP:
    $S_n = a + (a + d) + (a + 2d) + ... + [a + (n - 1)d]$
  - Write the sum again in reverse order:
    $S_n = [a + (n - 1)d] + [a + (n - 2)d] + ... + a$
  - Add the two expressions term by term:
    $2S_n = n[2a + (n - 1)d]$
  - Therefore, the sum of the first n terms is:
    $S_n = n/2 * [2a + (n - 1)d]$
- Explain each step clearly and emphasize why adding forward and backward sums works.
- **Examples and practice:**
  - Solve example problems with the class:
    Example 1: Find the sum of the first 10 terms of 3, 7, 11,...
    Example 2: Find the sum of first 15 terms of 20, 17, 14,...
- Let students try a few problems on their own or in pairs.

**3. Consolidation and Summary (15 minutes)**

- Recap the formula and its components.
- Ask some quick oral questions to check understanding.

- Highlight common mistakes to avoid.

**4. Assessment and Homework (5 minutes)**

- Give short questions to solve as homework:
    - Find the sum of the first 20 terms of 5, 8, 11,...
    - Find the sum of the first 12 terms of 50, 45, 40,...

**14. Factors to Consider When Purchasing Mathematics Reference Books**

1. **Relevance to the Curriculum:**
   The books should align with the national or school syllabus so that they cover the required topics appropriately.
2. **Level of Difficulty:**
   Books should be suitable for the students' level (Form Two in this case), ensuring content is neither too advanced nor too simple.
3. **Quality and Credibility:**
   The books should be written by reputable authors or publishers, preferably recognized in the educational field.
4. **Cost and Budget:**
   The price of the books should fit within the school's budget without compromising quality.
5. **Availability of Examples and Exercises:**
   Good reference books must have clear explanations, worked examples, and plenty of practice questions to aid understanding.