THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
DIPLOMA IN SECONDARY EDUCATTION EXAMINATION

740 **MATHEMATICS**

Time: 3 Hours **ANSWERS** **Year: 2024**

Instructions.

1. This paper consists of sections **A** and **B** with total of **Fourteen(14)** questions.

2. Answer **all** questions

3. Section **A** comprises **Ten (10)** questions with total of **40** marks, while section B has four questions with total of **60** marks..

4. Cellular phones are **note** allowed in the examination room.

5. Write your **examination Number** on every page of your answer booklet(s).

SECTION A (40 Marks)

Answer **all** questions from this section. Each question has **four (4)** marks.

1. Determine whether the given sentences are mathematical statements and give reasons:

   (a) The sentence "I did not pass the examination, did I?" is not a mathematical statement because it is a question. A mathematical statement must be a declarative sentence that is either true or false, and a question does not satisfy this condition.

   (b) The sentence "I passed the examination" is a mathematical statement. It is a declarative sentence that can be classified as either true or false depending on the factual situation.

   (c) The sentence "I entered in a classroom without permission" is a mathematical statement. It is a declarative sentence that is either true or false based on what happened.

   (d) The sentence "Please sir, may I enter in the classroom?" is not a mathematical statement because it is a request. A mathematical statement cannot be a command or a request; it must declare a fact or condition that can be true or false.

2. (a) Evaluate the integral $\int_0^1 x^2\,dx$ correct to 5 decimal places.

   $\int_0^1 x^2\,dx = [x^3/3]_0^1$

   $= (1^3/3) - (0^3/3)$

   $= 1/3$

   $= 0.33333$

   Therefore, the value is 0.33333.

   (b) Find the value of t correct to 5 decimal places, if s = 2t + W given that s = 8.235 and W = 4.365.

   $2t + 4.365 = 8.235$

   $2t = 8.235 - 4.365$

   $2t = 3.870$

   $t = 3.870 \div 2$

   $t = 1.93500$

   So, the value of t is 1.93500.

   (c) Find the value of the expression $\sqrt{(5 + 7)} \div 3$ correct to 5 decimal places.

   $\sqrt{(5 + 7)} \div 3 = \sqrt{12} \div 3$

   $= 3.46410 \div 3$

   $= 1.15470$

   Therefore, the value is 1.15470.

3. Locate the feasible region and determine corner points that would allow you to evaluate the objective function of the linear programming problem.

   Given:

   Maximize $f(x, y) = 18x + 10y$

   Subject to:

   $4x + y \leq 20$

   $2x + 3y \leq 30$

   $2x + y \geq 12$

First, draw the lines:
$4x + y = 20$
$2x + 3y = 30$
$2x + y = 12$
Find the points of intersection:
Intersection of $4x + y = 20$ and $2x + 3y = 30$:

Solve simultaneously:
$4x + y = 20$ (1)
$2x + 3y = 30$ (2)
From (1): $y = 20 - 4x$

Substitute into (2):
$2x + 3(20 - 4x) = 30$
$2x + 60 - 12x = 30$
$-10x = -30$
$x = 3$

Then $y = 20 - 4(3) = 8$
So, **point A (3, 8)**
Intersection of $2x + 3y = 30$ and $2x + y = 12$:
From $2x + y = 12$: $y = 12 - 2x$
Substitute into $2x + 3y = 30$:
$2x + 3(12 - 2x) = 30$
$2x + 36 - 6x = 30$
$-4x = -6$
$x = 1.5$
Then $y = 12 - 2(1.5) = 9$
So, **point B (1.5, 9)**
Intersection of $4x + y = 20$ and $2x + y = 12$:
Subtract the second from the first:
$(4x + y) - (2x + y) = 20 - 12$
$2x = 8$
$x = 4$
Then $y = 20 - 4(4) = 4$
So, **point C (4, 4)**
Now, test these points in $f(x, y) = 18x + 10y$:
At (3,8): $18(3) + 10(8) = 54 + 80 = 134$
At (1.5,9): $18(1.5) + 10(9) = 27 + 90 = 117$
At (4,4): $18(4) + 10(4) = 72 + 40 = 112$
Maximum value is 134 at (3, 8)

4. Outline the steps that a teacher has to follow when guiding students to understand the theorem that states that, “an angle in a semi-circle is a right angle.”
   - The teacher begins by explaining the statement of the theorem and its meaning using simple language and practical situations.
   - Next, the teacher draws a circle with a diameter and selects a point on the circumference to form a triangle using the diameter as the base.
   - Then, the teacher measures the angle formed at the circumference opposite the diameter and shows students that it is 90°.
   - Finally, the teacher leads students through the proof of the theorem using known properties of angles in a triangle and cyclic quadrilaterals.

5. A shadow of an object is cast into a wall. Find the position of the object so that the length of the shadow is twice the length of the object.

   Let the length of the object be h and the distance from the light source to the object be x. The distance from the object to the wall is y.
   Given: $y = 2h$
   Using similar triangles:
   $(h / x) = (2h / (x + y))$
   Simplifying:
   $1 / x = 2 / (x + y)$
   Cross multiply:
   $x + y = 2x$
   Then $y = x$
   Since $y = 2h$, then $x = 2h$.

   Therefore, the position of the object is at a distance of 2h from the light source.

6. A boat is sailing directly towards a cliff. The angle of elevation of a point on the top of a cliff and straight ahead of the boat increases from 10° and 15° as the boat sails at a distance of 50 m. Find the height of the cliff, approximately to one decimal place.
   Let h be the height of the cliff and d be the initial distance from the cliff.
   Using $\tan \theta = h/d$:
   At 10°: $\tan(10°) = h / (d + 50)$
   At 15°: $\tan(15°) = h / d$
   From these:
   $h = d \times \tan(15°)$
   $h = (d + 50) \times \tan(10°)$
   Equating:
   $d \times \tan(15°) = (d + 50) \times \tan(10°)$
   $d (\tan(15°) - \tan(10°)) = 50 \times \tan(10°)$
   Substitute:
   $\tan(15°) \approx 0.26795$
   $\tan(10°) \approx 0.17633$

$d = (50 \times 0.17633) / (0.26795 - 0.17633)$
$= 8.8165 / 0.09162$
≈ 96.27 m
Then $h = d \times \tan(15°)$
$= 96.27 \times 0.26795$
≈ 25.8 m
Therefore, the height of the cliff is approximately 25.8 m.

7. Prove that $\tanh^{-1}((x^2 - 1) / (x^2 + 1)) = \ln x$

Proof:
We start by recalling the definition of the inverse hyperbolic tangent:
$\tanh^{-1} y = 0.5 \times \ln((1 + y) / (1 - y))$
Now, let's let
$y = (x^2 - 1) / (x^2 + 1)$
Then, applying the formula:
$\tanh^{-1} y = 0.5 \times \ln((1 + y) / (1 - y))$
Substituting y:
$= 0.5 \times \ln((1 + (x^2 - 1) / (x^2 + 1)) / (1 - (x^2 - 1) / (x^2 + 1)))$

Simplify the numerators and denominators inside the brackets:
$= 0.5 \times \ln(((x^2 + 1) + (x^2 - 1)) / ((x^2 + 1) - (x^2 - 1)))$

Simplify both numerators:
Numerator: $(x^2 + 1) + (x^2 - 1) = 2x^2$
Denominator: $(x^2 + 1) - (x^2 - 1) = 2$

So now:
$= 0.5 \times \ln(2x^2 / 2)$

Simplify the fraction:
$= 0.5 \times \ln(x^2)$
And since $\ln(x^2) = 2\ln x$:
$= 0.5 \times (2\ln x)$

Simplify:
$= \ln x$
Hence proved.

8. Outline four curriculum materials that are applied in teaching and learning of Mathematics.
   - Mathematics textbooks containing detailed explanations, exercises, and worked examples for students' reference and teachers' guidance.
   - Mathematics syllabus which outlines the topics, objectives, and assessment methods to be covered at each education level.
   - Mathematical instruments such as rulers, protractors, compasses, and set squares used for practical geometry and measurements.

- Visual aids like charts, graphs, and models that help in illustrating mathematical concepts and enhancing learners' understanding.

9. If vectors a = (3, t) and b = (t, 4) are perpendicular, find the value(s) of t.

   Vectors are perpendicular if their dot product is zero.
   $a \cdot b = (3 \times t) + (t \times 4) = 0$
   $3t + 4t = 0$
   $7t = 0$
   $t = 0$
   Therefore, the value of t is 0.

10. Briefly explain four properties of teaching and learning aids for effective understanding of learners.
    - Teaching aids should be clear and visible to all learners in the classroom to ensure that every student can benefit from them.
    - They should be relevant to the topic being taught, directly supporting the intended lesson objectives.
    - Teaching aids must be durable and safe for repeated use without posing harm to learners during handling.
    - They should be simple and easy to use, avoiding complicated or overly technical features that might confuse learners.

SECTION B (60 Marks)

Answer **all** questions from this section. Each question has **fifteen (15)** marks.

11. (a) Identify type of the conic section $2x^2 - 4x - 4y - 2 = 0$ by its centre, focus, and axis.

Let's first rewrite the given equation:

$2x^2 - 4x - 4y - 2 = 0$

Divide through by 2:

$x^2 - 2x - 2y - 1 = 0$

Group x-terms:

$(x^2 - 2x) = 2y + 1$

Complete the square for x:

$= (x - 1)^2 - 1 = 2y + 1$

Simplify:

$(x - 1)^2 = 2y + 2$

Then:

$(x - 1)^2 = 2(y + 1)$

This is the standard form of a **parabola** opening upwards.

Centre (vertex) is at (1, –1)

Focus: From $(x – h)^2 = 4p(y – k)$

Comparing: $4p = 2$

$p = 0.5$

Focus is at $(1, –1 + 0.5) = (1, –0.5)$

Axis of symmetry is the line $x = 1$

(b) Sketch the graph of $(x – 1)^2 = 2(y + 1)$, hence label the vertices, co-vertices and centre.

Since this is a parabola, it has no co-vertices.

Centre (vertex) is $(1, –1)$

Focus is $(1, –0.5)$

Directrix is $y = –1.5$

The parabola opens upwards.

Key points:

Vertex $(1, –1)$

Focus $(1, –0.5)$

No co-vertices for a parabola.

12. (a) (i) A committee consisting of 5 men and 6 women is to be chosen from 7 men and 9 women. In how many ways can this be done?

Number of ways to choose 5 men from 7:

= 7C5 = 21

Number of ways to choose 6 women from 9:

= 9C6 = 84

Total ways:

$= 21 \times 84 = 1764$ ways

(ii) A box contains 2 yellow, 5 red, and 4 green marbles. How many red marbles should be taken so that probability of taking one green is 21?

Total marbles = 2 + 5 + 4 = 11

If x red marbles are taken out:

Remaining marbles = 11 – x

Now, probability of taking one green = 4 / (11 – x) = 2 / 1

(b) The probability that a man will have a capital given he started a business is 0.4

$P(C \mid B) = 0.4$

The probability he will start the business given he has capital is 0.25

$P(B \mid C) = 0.25$

The probability that he will have capital and start a business is 0.12

$P(C \cap B) = 0.12$

(i) Find P(B)

Using conditional probability:

$P(C \cap B) = P(C \mid B) \times P(B)$

$0.12 = 0.4 \times P(B)$

$P(B) = 0.12 / 0.4$

$P(B) = 0.3$

(ii) Probability that he will have capital and not start business

$P(C \cap B') = P(C) - P(C \cap B)$

First, find P(C) using:

$P(C \cap B) = P(B \mid C) \times P(C)$

$0.12 = 0.25 \times P(C)$

$P(C) = 0.12 / 0.25$

$P(C) = 0.48$

Now:

$P(C \cap B') = 0.48 - 0.12$

$= 0.36$

13. In order to improve teaching and learning of Mathematics, teachers should be equipped with assessment tools. Analyse six assessment tools that you can use to assess students understanding of Mathematics.

One tool is **written tests and examinations**. These evaluate students' ability to solve problems, explain concepts, and apply mathematical formulas within a set time.

Another tool is **oral questioning**, where teachers ask students verbal questions during or after lessons to check comprehension and correct misconceptions immediately.

Assignments and homework provide opportunities for students to practice independently, and help teachers assess long-term retention and understanding.

Project work requires students to apply mathematical concepts in real-life contexts, encouraging creativity, teamwork, and critical thinking.

Observation checklists allow teachers to assess students' participation, attitude, and problem-solving approaches during class activities.

Peer assessment involves students reviewing each other’s work, which helps them learn by teaching and identifying errors in reasoning.

14. Competence on the principles of teaching and learning of Mathematics. Explain six principles of teaching and learning of Mathematics.

The first principle is **from known to unknown**. Teachers should begin with concepts familiar to students and gradually introduce new ideas, building on existing knowledge.

The second principle is **activity-based learning**. Mathematics should be taught through practical activities and problem-solving to help students actively construct knowledge.

The third principle is **logical sequence**. Topics must be arranged progressively from simple to complex so that each new concept builds logically on the previous.

Another principle is **practice and repetition**. Regular practice enables students to master mathematical techniques, strengthen memory, and develop problem-solving skills.

Use of teaching aids and materials is essential. Visuals like models, charts, and diagrams simplify abstract ideas and improve learners’ comprehension.

Lastly, **assessment and feedback** are crucial. Teachers should continuously assess learners’ progress and provide feedback, which helps identify difficulties and guide future learning.