THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
FORM TWO NATIONAL ASSESSMENT

042 **ADDITIONAL MATHEMATICS**

Time: 2:30 Hours **SOLUTIONS** **Year: 2018**

Instructions

1. This paper consists of two sections of **ten (10) Compulsory** questions.
2. Answer **all** questions.
3. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
4. Cellular phones and any unauthorized materials are **not** allowed in the assessment room.
5. Write your **Assessment Number** at the top right hand corner of every page.

1. (a) Write the first four multiples of 17.

The first four multiples of 17 are **17, 34, 51 and 68**. These are obtained by multiplying 17 by 1, 2, 3 and 4.

(b) Identify the numbers which are divisible by 3 among 8476, 942, 5181, 7124, 35768 and 91284.

A number is divisible by 3 if the sum of its digits is divisible by 3.

The sum of digits of 8476 is 8+4+7+6=25. Since 25 is not divisible by 3, the number is not divisible by 3.

The sum of digits of 942 is 9+4+2=15. Since 15 is divisible by 3, the number is divisible by 3.

The sum of digits of 5181 is 5+1+8+1=15. Since 15 is divisible by 3, the number is divisible by 3.

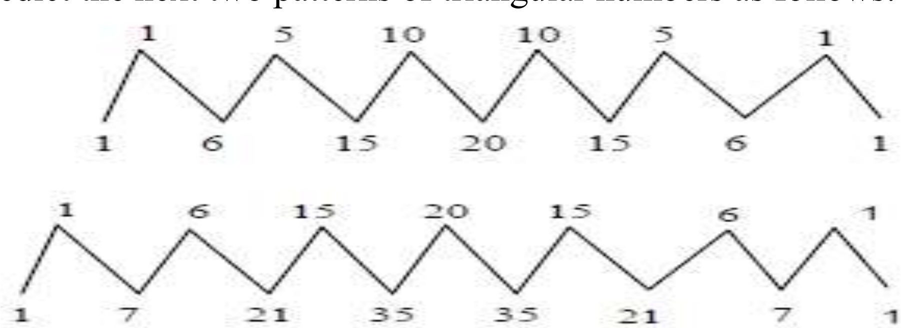
The sum of digits of 7124 is 7+1+2+4=14. Since 14 is not divisible by 3, the number is not divisible by 3.

The sum of digits of 35768 is 3+5+7+6+8=29. Since 29 is not divisible by 3, the number is not divisible by 3.

The sum of digits of 91284 is 9+1+2+8+4=24. Since 24 is divisible by 3, the number is divisible by 3.

Therefore, Numbers which are divisible by 3 are 942, 5181, and 91284

(c) Predict the next two patterns of triangular numbers as follows:



2. (a) Make b the subject from the formula 1/a + 1/b = 1/c

Start by subtracting 1/a from both sides to get 1/b = 1/c − 1/a. Then find a common denominator so 1/b = (a − c)/(ac). Taking the reciprocal gives

b = ac/(a − c).

(b) Determine the value of n by using the formula c = (nE)/(R + nr), where c = 1.5, E = 3, R = 6.3 and r = 1.1

Substitute values to get 1.5 = (3n)/(6.3 + 1.1n).

Multiply both sides by the denominator

1.5(6.3 + 1.1n) = 3n.

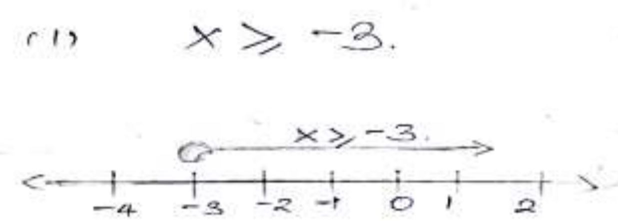
9.45 + 1.65n = 3n.

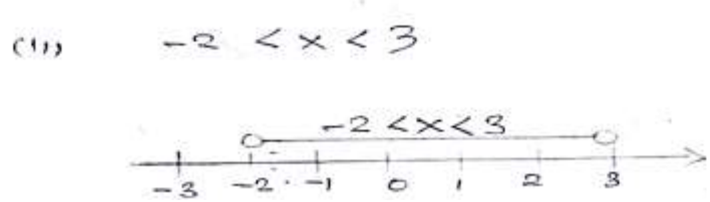
9.45 = 1.35n.

n = 9.45/1.35

n = 7.

(c) Show the following inequalities on a number line





3. (a) Find the sum of interior angles of regular pentagon and hence determine the size of each interior angle.

The sum of interior angles of an n sided polygon is $(n - 2) \times 180^0$.

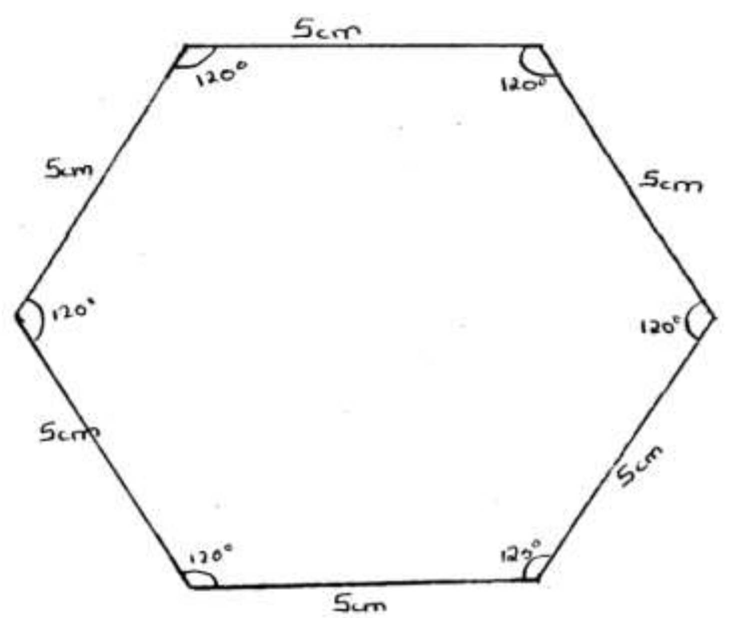
For a pentagon n = 5, so

$S = (5 - 2) \times 180^0 = \mathbf{540^0}$

Size of each interior angle = $540^o / 5 = 108^0$

Size of each interior angle is 108^0

(b) Draw a regular hexagon with 5 cm each side.



4. (a) Find the equation of locus of the points which have equal distance from points A $(-5, 8)$ and B $(6, 7)$.

Given points $A(-5, 8)$ and $B(6, 7)$

Distance PA squared = $(x + 5)^2 + (y - 8)^2$
Distance PB squared = $(x - 6)^2 + (y - 7)^2$

Set them equal:
$(x + 5)^2 + (y - 8)^2 = (x - 6)^2 + (y - 7)^2$

Expand left:
$x^2 + 10x + 25 + y^2 - 16y + 64$

Expand right:
$x^2 - 12x + 36 + y^2 - 14y + 49$

Simplify:
$x^2 + 10x + 25 + y^2 - 16y + 64 = x^2 - 12x + 36 + y^2 - 14y + 49$

Cancel x^2 and y^2:
$10x - 16y + 89 = -12x - 14y + 85$

Add 12x to both sides:
$22x - 16y + 89 = -14y + 85$

Add 14y:
$22x - 2y + 89 = 85$

Subtract 89:
$22x - 2y = -4$

Divide by 2:
$11x - y = -2$

Equation of the locus:
$11x - y = -2$

(b) State the equation of the locus of point P which its distance from point A (−1,−3) is twice its distance from point B (2,4).

Point P(x, y) is such that PA = 2 PB

Distance squared:

$PA^2 = (x + 1)^2 + (y + 3)^2$

$PB^2 = (x - 2)^2 + (y - 4)^2$

Since PA = 2 PB:

$(x + 1)^2 + (y + 3)^2 = 4[(x - 2)^2 + (y - 4)^2]$

Expand left:

$x^2 + 2x + 1 + y^2 + 6y + 9 = x^2 + y^2 + 2x$?

Left: $x^2 + 2x + 1 + y^2 + 6y + 9 = x^2 + y^2 + 2x + 6y$

Left: $(x + 1)^2 = x^2 + 2x + 1$

$(y + 3)^2 = y^2 + 6y + 9$

Sum $= x^2 + y^2 + 2x + 6y + 10$

Right: $4[(x-2)^2 + (y-4)^2] = 4[x^2 - 4x + 4 + y^2 - 8y + 16] = 4x^2 - 16x + 16 + 4y^2 - 32y + 64 = 4x^2 + 4y^2 - 16x - 32y + 80$

Now equation:

$x^2 + y^2 + 2x + 6y + 10 = 4x^2 + 4y^2 - 16x - 32y + 80$

Bring all terms to left:

$x^2 - 4x^2 + y^2 - 4y^2 + 2x + 16x + 6y + 32y + 10 - 80 = 0$

$-3x^2 -3y^2 +18x +38y -70 = 0$

Multiply through by −1:

$3x^2 +3y^2 -18x -38y +70 = 0$

Divide whole equation by 1 (or 3 to simplify):

$x^2 + y^2 -6x -(38/3)y + 70/3 = 0$

Equation of locus:

$x^2 + y^2 - 6x - (38/3)y + 70/3 = 0$

5. (a) Find the value of r from the line joining point P(r,3) to the point Q(2,−3) which is perpendicular to the line joining point R(10,1) to point Q.

Given: Line PQ is perpendicular to line RQ.

Points: P(r, 3), Q(2, −3), R(10, 1)

Find slope of RQ

Slope of RQ = $(y_Q - y_R) / (x_Q - x_R) = (-3 - 1)/(2 - 10) = (-4)/(-8) = 1/2$

Slope of PQ (perpendicular)

Slope of PQ × slope of RQ = −1 (perpendicular condition)

Let slope of PQ = $(-3 - 3)/(2 - r) = (-6)/(2 - r)$

Set product with slope of RQ = −1:

$(-6)/(2 - r) \times (1/2) = -1$

Simplify:

$(-6)/[2(2 - r)] = -1$

$(-6)/(4 - 2r) = -1$

Multiply both sides:

$-6 = -(4 - 2r)$

$-6 = -4 + 2r$

$-6 + 4 = 2r$

$-2 = 2r$

$r = -1$

Answer: $r = -1$

(b) Find the equation of a line passing through the point $(1,-3)$ which is parallel to the line $2x + 3y - 4 = 0$.

Substitute the point to get $2(1) + 3(-3) + k = 0$

$-7 + k = 0$ so $k = 7$.

Therefore the required line is $2x + 3y + 7 = 0$.

6. (a) Find the number of lines of symmetry for the following figures: (i) equilateral triangle (ii) a square

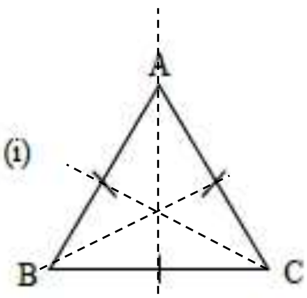
   (i) An equilateral triangle has three lines of symmetry.

   (ii) A square has four lines of symmetry.

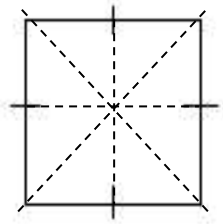
(b) Draw the lines of symmetry on the figures in part (a).

For the triangle, each line goes from a vertex to the midpoint of the opposite side. For the square, two lines go through opposite vertices and two lines go through midpoints of opposite sides.

(i)



(ii)



7. (a) Let p be "He is tall" and q be "He is handsome". Write the following statements in symbolic form:

(i) He is tall or he is short and handsome. Symbolically **$p \vee (\neg p \wedge q)$.**

(ii) It is not true that he is short or not handsome. Symbolically **$\neg(\neg p \vee \neg q)$.**

(iii) He is handsome if and only if he is tall. Symbolically **$q \leftrightarrow p$.**

(iv) If he is handsome then he is either tall or short. Symbolically **$q \rightarrow (p \vee \neg p)$.**

(b) (i) Construct a truth table of the compound proposition $(p \vee q) \wedge (\neg p \vee q)$.

p	q	~p	p∨q	~p∨q	(p∨q)∧(~p∨q)
T	T	F	T	T	T
T	F	F	T	F	F
F	T	T	T	T	T
F	F	T	F	T	F

(ii) Show whether the logical statement (p→q)∨(q→p) is a tautology or not.

p	q	p→q	q→p	(p→q)∨(q→p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The statement is always true for all possible truth values so it is a tautology.

8. (a) Given that variable y varies jointly as x and z. If y = 10, x = 4 and z = 5, find the value of z when x = 2, and y = 5

Joint variation means y = kxz.

When y = 10, x = 4, z = 5, k = ?

k = 10/(5 x 4) = 0.5

When x = 2, y = 5, z = ?

z = y/(kx) = (5 x 2)/2 = 5

z = 5

(b) If 2 students can type 210 pages in 3 days, find the number of students that are needed to type 700 pages in 2 days.

Given: 2 students → 210 pages in 3 days

Find pages per student per day

Total student-days = 2 × 3 = 6 student-days

Pages per student per day = 210 ÷ 6 = **35 pages/day**

Find total student-days needed for 700 pages

Total pages needed = 700

Pages per student per day = 35

Total student-days = 700 ÷ 35 = **20 student-days**

Find number of students for 2 days

Let number of students = n

n × 2 days = 20 student-days

n = 20 ÷ 2 = 10

10 students are needed to type those pages

9. In a group of 450 students; 100 play volleyball, 70 play athletics, 200 play drama, 90 play volleyball and participate in drama, 30 play volleyball and athletics, 45 athletes and participate in drama, 220 do not participate in any game. Use the general formula of union of sets to find the number of participants in all games.

Given,

450 students in total

100 play volleyball

70 play athletics

200 play drama

90 play volleyball and drama

30 play volleyball and athletics

45 play athletics and drama

220 do not participate in any game

Let V = volleyball, A = athletics, D = drama

From the formula, $n(V \cup A \cup D) = n(V) + n(A) + n(D) - n(V \cap A) - n(A \cap D) - n(V \cap D) + n(V \cap A \cap D)$

$450 = 100 + 70 + 200 - 30 - 45 - 90 + n(V \cap A \cap D) + 220$

$450 = 370 - 30 - 45 - 90 + 220 + n(V \cap A \cap D)$

$450 = 370 + 220 - 30 - 45 - 90 + n(V \cap A \cap D)$

$450 = 590 - 30 - 45 - 90 + n(V \cap A \cap D)$

$450 = 560 - 45 - 90 + n(V \cap A \cap D)$

$450 = 515 - 90 + n(V \cap A \cap D)$

$450 = 425 + n(V \cap A \cap D)$

$n(V \cap A \cap D) = 450 - 425$

$n(V \cap A \cap D) = 25$

The number of students who participate in all the games is 25

10. Solve the following set of equations simultaneously using substitution method: $xy = 64$, $4x - y = 60$

Given:

$xy = 64$ --- (i)

$4x - y = 60$ --- (ii)

From (i), make x the subject:

$x = 64 / y$ --- (iii)

Substitute (iii) into (ii):

$4(64 / y) - y = 60$

$256 / y - y = 60$

Multiply through by y:

$256 - y^2 = 60y$

$-y^2 - 60y + 256 = 0$

Quadratic: $-y^2 - 60y + 256 = 0$

$a = -1$, $b = -60$, $c = 256$

Use quadratic formula:

$y = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

$y = [60 \pm \sqrt{(3600 + 1024)}] / -2$

$y = [60 \pm \sqrt{4624}] / -2$

$y = [60 \pm 68] / -2$

So:

$y = (60 + 68)/-2 = 128/-2 = -64$

$y = (60 - 68)/-2 = -8/-2 = 4$

Substitute into $x = 64 / y$:

If $y = -64$, $x = 64 / -64 = -1$

If $y = 4$, $x = 64 / 4 = 16$

Answer:

$x = -1$ or 16, $y = -64$ or 4