THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
FORM TWO NATIONAL ASSESSMENT

042 **ADDITIONAL MATHEMATICS**

Time: 2:30 Hours **SOLUTIONS** **Year: 2022**

Instructions

1. This paper consists of two sections of **ten (10) Compulsory** questions.
2. Answer **all** questions.
3. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
4. Cellular phones and any unauthorized materials are **not** allowed in the assessment room.
5. Write your **Assessment Number** at the top right hand corner of every page.

Prepared by Maria Marco for TETEA

1. (a) Determine the first 5 prime numbers of the Fibonacci sequence

   Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...

   Prime numbers from this list: 2, 3, 5, 13, 89

   First 5 primes: 2, 3, 5, 13, 89

   (b)(i) Find the next four numbers in the sequence 1, 4, 10, 22

   Pattern: multiply by increasing integers

   next numbers are 46, 94, 190 and 382

   (ii) Check divisibility of 9142

   Divisible by 5? Last digit 2, so no.

   Divisible by 6? Must be divisible by 2 and 3.

   Divisible by 2? Yes.

   Divisible by 3? Sum = 9+1+4+2 = 16. 16 not divisible by 3. So 9142 not divisible by 6.

   Divisible by 7? Perform 9142 divided by 7 = 1306, divisible.

   Divisible by 9? Sum of digits = 16. Not divisible by 9.

   Final: Not divisible by 5, 6, and 9., but divisible by 7

2. (a) Given that A/b = sqrt((f+p)/(f-p)), express f in terms of A, b and p.

   Given:

   $A / b = \sqrt{(f + p) / (f - p)}$

   Solution:

Step 1: Square both sides to remove the square root:

$(A / b)^2 = (f + p) / (f - p)$

Step 2: Multiply both sides by (f - p):

$(f - p) \times (A^2 / b^2) = f + p$

Step 3: Expand the left-hand side:

$(A^2 / b^2) \times f - (A^2 / b^2) \times p = f + p$

Step 4: Bring all terms containing f to one side and constants to the other:

$(A^2 / b^2) \times f - f = p + (A^2 / b^2) \times p$

Step 5: Factor f:

$f \times [(A^2 / b^2) - 1] = p \times [1 + (A^2 / b^2)]$

Step 6: Solve for f:

$f = [p \times (1 + A^2 / b^2)] / [(A^2 / b^2) - 1]$

Step 7: Simplify numerator and denominator:

$f = p \times [(b^2 + A^2) / b^2] \div [(A^2 - b^2) / b^2]$

$f = p \times (A^2 + b^2) / (A^2 - b^2)$

$\mathbf{f = p \times (A^2 + b^2) / (A^2 - b^2)}$

(b) Solve the simultaneous equations:

$p/5 + (2q)/3 = 49/15$

$(3p)/7 - q/2 + 5/7 = 0$

$p/5 + (2q)/3 = 49/15$

$3p/7 - q/2 + 5/7 = 0$

Step 1: Eliminate fractions

Equation 1: $p/5 + 2q/3 = 49/15$

The LCM of denominators 5 and 3 is 15. Multiply entire equation by 15:

$15 \times (p/5) + 15 \times (2q/3) = 15 \times (49/15)$

$3p + 10q = 49$ …(1)

Equation 2: $3p/7 - q/2 + 5/7 = 0$

The LCM of 7 and 2 is 14. Multiply entire equation by 14:

$14 \times (3p/7) - 14 \times (q/2) + 14 \times (5/7) = 0$

$6p - 7q + 10 = 0$

$6p - 7q = -10$ …(2)

Step 2: Solve using elimination

Equation (1): $3p + 10q = 49$

Equation (2): $6p - 7q = -10$

Multiply equation (1) by 2 to align p-terms:

$6p + 20q = 98$

Now subtract equation (2):

$(6p + 20q) - (6p - 7q) = 98 - (-10)$

$6p + 20q - 6p + 7q = 108$

$27q = 108$

$q = 108 / 27$

$q = 4$

Step 3: Solve for p

Substitute $q = 4$ into equation (1):

$3p + 10(4) = 49$

$3p + 40 = 49$

$3p = 49 - 40$

$3p = 9$

$p = 9 / 3$

$p = 3$

$p = 3, q = 4$

3. (a) Sum of interior angles = 1440^o

   Formula: $(n - 2) \times 180 = 1440^0$

   $n - 2 = 1440^0/180$

   $n - 2 = 8$

   $n = 10$

(i) Number of sides: 10

(ii) Name: Decagon

(iii) Maximum triangles = n - 2 = 8

(b) Exterior angle is 30 degrees less than half the interior angle

Let interior angle = I

Exterior = 180 - I

Given:

180 - I = (I/2) - 30

Solve:

180 - I = I/2 - 30

Add I to both sides:

180 = I/2 + I - 30

180 = 1.5I - 30

Add 30:

210 = 1.5I

I = 210 / 1.5

I = 140 degrees

Exterior = 180 - 140 = 40 degrees

Number of sides = 360 / exterior = 360 / 40 = 9

4. (a) Locus of Q(x,y)

Distance to (3,4) equals twice distance to line x = 3

Distance to point: sqrt((x - 3)^2 + (y - 4)^2)

Distance to line: |x - 3|

Equation:

sqrt((x - 3)^2 + (y - 4)^2) = 2|x - 3|

Square:

(x - 3)^2 + (y - 4)^2 = 4(x - 3)^2

the locus is $(y - 4)^2 = 3(x - 3)^2$

(b) Locus of P(x,y)

Point S = (2, -3)

Find intersection of lines:

Solve x - 2y = 4

2x + 3y = 15

Multiply first by 3:

3x - 6y = 12

Second by 2:

4x + 6y = 30

Add: 7x = 42

x = 6

Substitute into x - 2y = 4:

6 - 2y = 4

2y = 2

$y = 1$

Intersection point is (6,1)

Distance to (2,-3) = distance to (6,1)

sqrt((x - 2)^2 + (y + 3)^2) = sqrt((x - 6)^2 + (y - 1)^2)

Square both sides:

(x - 2)^2 + (y + 3)^2 = (x - 6)^2 + (y - 1)^2

Expand:

x^2 - 4x + 4 + y^2 + 6y + 9 = x^2 - 12x + 36 + y^2 - 2y + 1

Combine:

-4x + 6y + 13 = -12x - 2y + 37

Add 12x to both sides:

8x + 6y + 13 = -2y + 37

Add 2y:

8x + 8y + 13 = 37

8x + 8y = 24

x + y = 3

Locus: x + y = 3

5. (a) Points A(8,10), B(5,n), C(0,2) are collinear

   Slope AB = slope BC

   (10 - n)/(8 - 5) = (n - 2)/(5 - 0)

(10 - n)/3 = (n - 2)/5

Cross multiply:

5(10 - n) = 3(n - 2)

50 - 5n = 3n - 6

50 + 6 = 8n

56 = 8n

n = 7

(b) Line through A(4,6), parallel to 2y = x - 2

Slope of given line: 1/2

Equation through (4,6):

y - 6 = 1/2(x - 4)

y - 6 = x/2 - 2

y = x/2 + 4

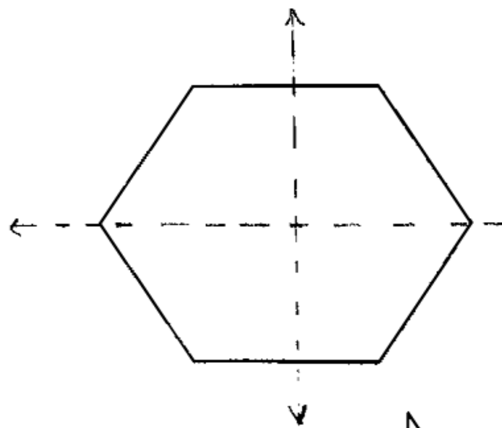
Find y intercept: put x=0

y = 4

Coordinates: (0,4)

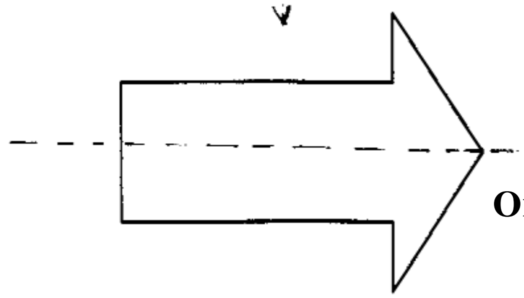
6. (a) Draw and state the number of the lins(s) of symmetry for each of the following figures.

(i)



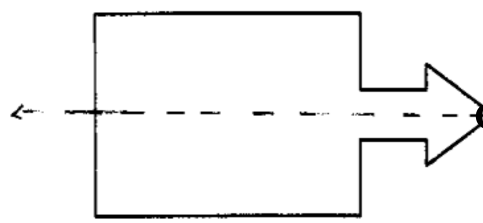
Two lines of symmetry

(ii)



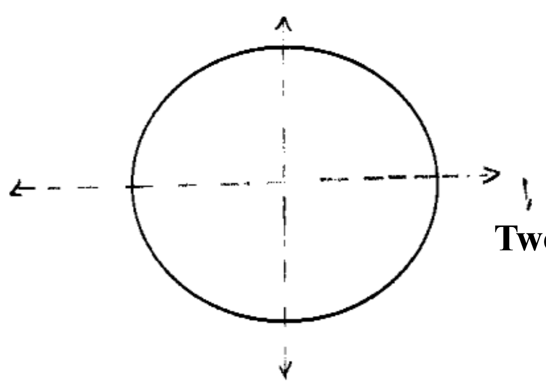
One line of symmetry

(iii)



One line of symmetry

(iv)



Two lines of symmetry

(b) Order of rotational symmetry

Circle: **infinite**

Equilateral triangle: **3**

Rectangle: **2**

Square: **4**

7. (a) Construct a truth table for each of the following propositions:

(i) (p^q)--→(pvq)

p	q	(p∧q)	(p∨q)	(p∧q) → (p∨q)
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

(ii) (-p^(p→q))-→-q

p	q	~p	~q	(p→q)	(~p∧(p→q))	(~p∧(p→q)) → ~q
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	F
F	F	T	T	T	T	T

(b) Use a truth table to test the validity of the argument “If it rains or no one comes, the game will not take place. The game was a success. Therefore, it did not rain”.

Let p = it rains, q = one comes, r = the game will not take place

$[((p \vee \sim q) \rightarrow \sim r) \wedge r) \rightarrow \sim p]$

to show Through The Truth Table

a, b

p	q	r	~p	~q	~r	(p∨~q)	(p∨~q)→~r	a∧r	b→~p
T	T	T	F	F	F	T	F	F	T
T	T	F	F	F	T	T	T	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	F	F	T	T	T
F	T	F	T	F	T	F	T	F	T
F	F	T	T	T	F	T	F	F	T
F	F	F	T	T	T	T	T	F	T

8. (a) x jointly proportional to y and 1/z

   $x = k(y/z)$

   Given x=3, y=2, z=1

   $3 = k(2/1)$

   $k = 3/2$

   Equation: $x = (3/2)(y/z)$

(ii) x = 6, y = 20

6 = (3/2)(20/z)

6 = 30/z

z = 30/6

z = 5

(b) Twelve men working for 10 hours per day take 4 weeks to plant maize in a certain farm. For how long should 20 men work per day in order to plant maize in the same farm for 14 weeks?

Let M, H and W for men, hour and week, respectively.

Then, m = K/WH

K = 120 X 4 = 480

So, H x 20 x 2 = 480

H = 12

It takes 12 hours

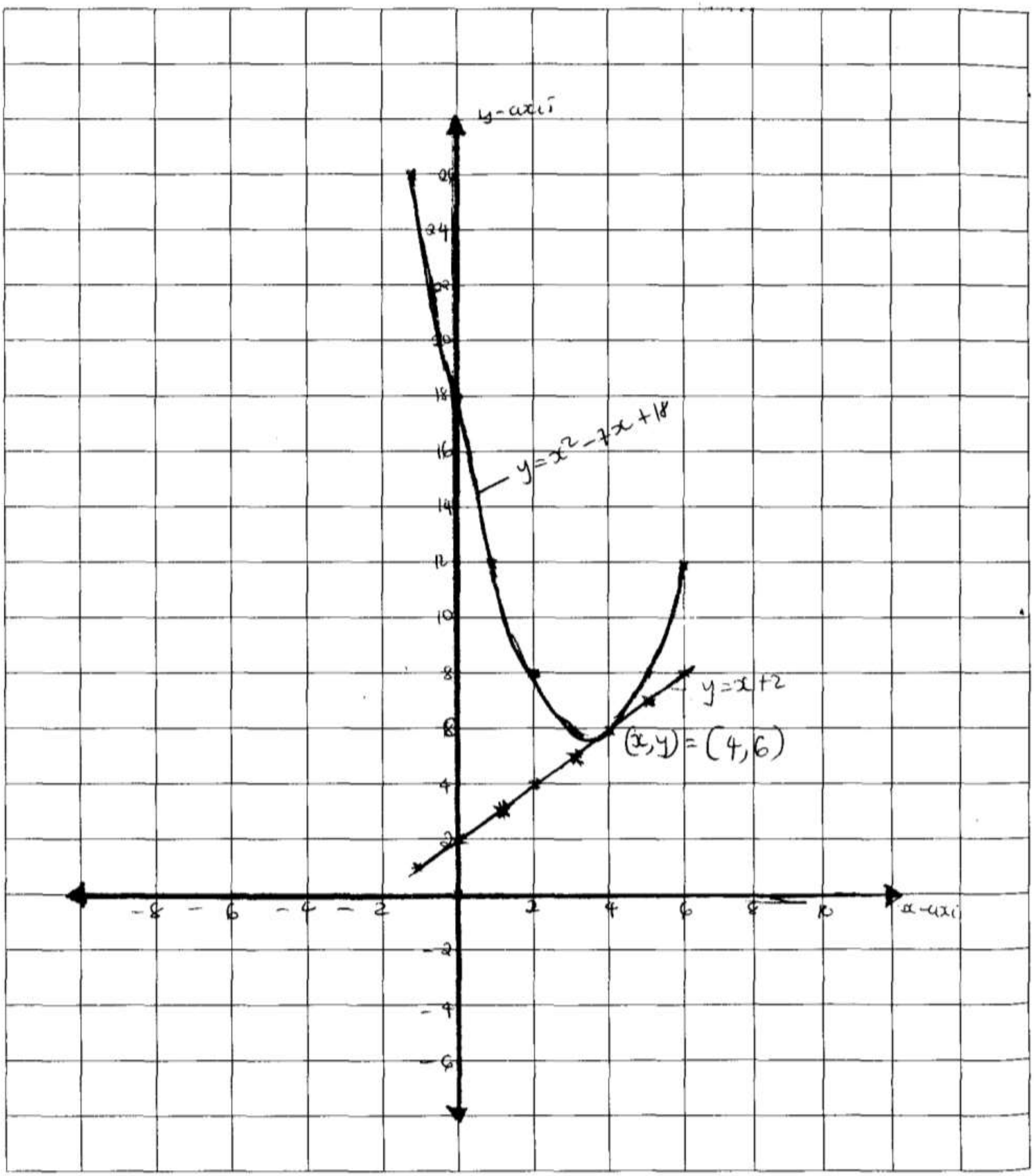
9. Draw the graphs of $y = x^2 - 7x + 18$ and y = x + 2 for -1<=x<=6 on the same xy-plane in the following graph space. Hence use the obtained graph to find the common solution of the given equation.

   Diven,

   $y = x^2 - 7x + 18$; y = x + 2

   Table of values:

x	-1	0	1	2	3	4	5	6
x+2	1	2	3	4	5	6	7	8
$x^2 - 7x + 18$	26	18	12	8	6	6	8	12



y-axis
x-axis
$y = x^2 - 7x + 18$
$y = x + 2$
$(x, y) = (4, 6)$

10. In a class of 63 students, 22 students study Biology, 26 study Chemistry, 25 study Physics, 18 study both Physics and Chemistry, 4 study both Biology and Chemistry, 3 study both Physics and Biology and 1 student studies all the three subjects.

(a) Represent this information using a Venn diagram.

Given:

Total students = 63

Biology = 22

Chemistry = 26

Physics = 25

Physics & Chemistry = 18

Biology & Chemistry = 4

Physics & Biology = 3

All three = 1

Step 1: Subtract students in all three from each pair:

Physics & Chemistry only = 18 - 1 = 17

Biology & Chemistry only = 4 - 1 = 3

Physics & Biology only = 3 - 1 = 2

Step 2: Students taking only one subject:

Biology only = 22 - (3 + 3 + 1) = 15

Chemistry only = 26 - (3 + 17 + 1) = 5

Physics only = 25 - (2 + 17 + 1) = 5

Step 3: Students taking none = Total - sum of all subsets:

Sum of all: 15 + 5 + 5 + 2 + 3 + 17 + 1 = 48

Students taking none = 63 - 48 = 15

Venn diagram values:

Biology only: 15

Chemistry only: 5

Physics only: 5

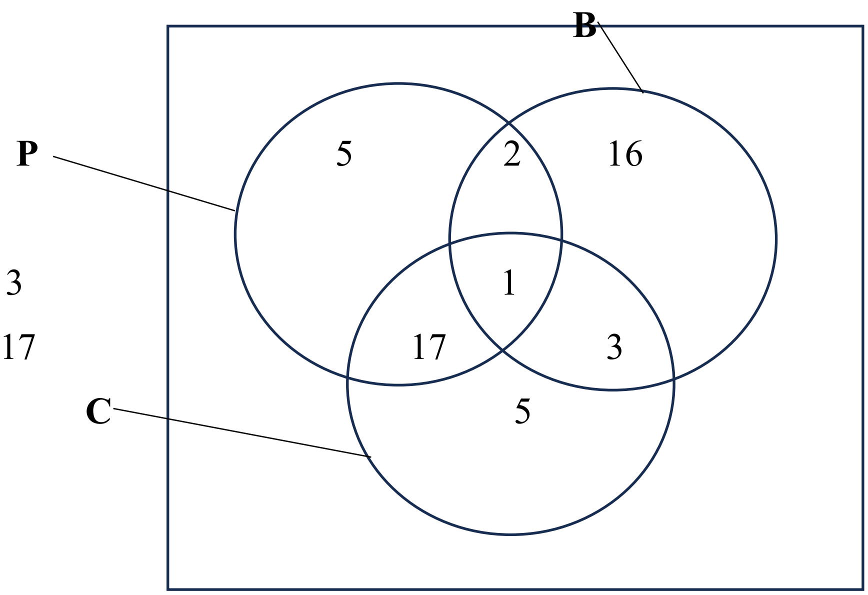
Biology & Chemistry: 3

Physics & Chemistry: 17

Physics & Biology: 2

All three: 1

None: 15



(b) Use the results of part (a) to determine the number of students who study:

(i) Biology only. **16**

(ii) Physics or Chemistry. **33**

(iii) None of the three subjects. **14**

(iv) Physics but not Chemistry. **7**