THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
FORM TWO NATIONAL ASSESSMENT

042 **ADDITIONAL MATHEMATICS**

Time: 2:30 Hours **SOLUTIONS** **Year: 2023**

Instructions

1. This paper consists of two sections of **ten (10) Compulsory** questions.
2. Answer **all** questions.
3. All writing must be in **blue** or **black** ink **except** drawing which must be in pencil.
4. Cellular phones and any unauthorized materials are **not** allowed in the assessment room.
5. Write your **Assessment Number** at the top right hand corner of every page.

1. (a) (i) Study the sequence -3, -2, -5, -7, -12, -19, and then state a reason to verify that the sequence is Fibonacci.

-3 + -2 = -5

-2 + -5 = -7

-5 + -7 = -12

-7 + -12 = -19

The sequence follows the Fibonacci rule.

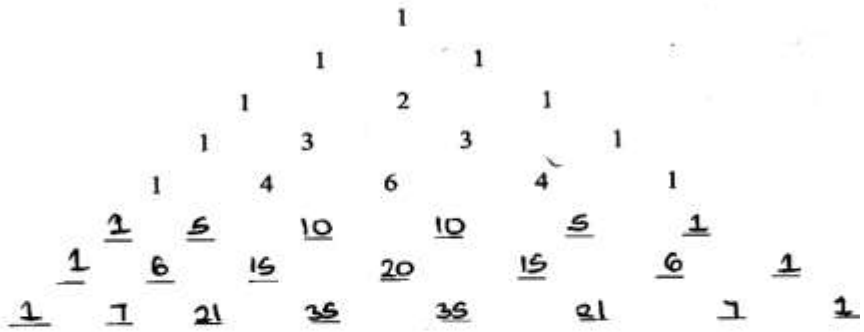
(ii) Use the divisibility rule to determine whether the number 9655 is divisible by 3.

Sum of digits = 9 + 6 + 5 + 5 = 25

25 is not divisible by 3

9655 is not divisible by 3

(b) Complete the blank spaces in the following pattern of numbers that obey Pascal's triangle.



2. (a) Simplify the expression 18r - (2r + 10) – 14r + 25 to its lowest term.

   18r - 2r - 10 - 14r + 25

   2r - 10 + 25

   2r + 15

   (b) Expand completely the following expressions:

   (i) $3(2c + 3)^2 – c^2$

   $3(4c^2 + 12c + 9) - c^2$

   $12c^2 + 36c + 27 - c^2$

   $11c^2 + 36c + 27$

   (ii) $2x(x + 4y) – x(8x + 14y) – 2(3 + 4y)$

   $2x^2 + 8xy - 8x^2 - 14xy - 6 - 8y$

   $-6x^2 - 6xy - 8y - 6$

   (c) Write r in terms of x and y, given that:

   $x/y = (1+r^2)/(1-r^2)$

   $= r = \sqrt{\frac{x-y}{x+y}}$

3. The size of an exterior angle of a certain polygon is p and the size of its interior angle is three times the size of the exterior angle.

   (i) Interior angle = **3p**

   (ii) p + 3p = 180

   4p = 180

p = 45°

Interior angle = 3 × 45 = **135°**

(iii) Number of sides n = 360 / exterior angle = 360 / 45 = 8

Sum of interior angles = (n - 2) × 180 = (8 - 2) × 180 = **1080°**

4. (a) Describe the locus when:

(i) An orange is falling vertically from a tree at a height of 2 metres from the ground

Locus: **vertical line along x = constant, y ≤ 2**

(ii) The centre of a wheel as a cyclist rides along the road on a horizontal plane

Locus: straight horizontal line along the road at the height of the wheel centre

(b) Analyse the locus of the point P which is equidistant from the points L(-2,2) and M(1, 1.5)

Midpoint = ((-2 + 1)/2 , (2 + 1.5)/2) = (-0.5, 1.75)

Slope LM = (1.5 - 2)/(1 + 2) = -0.5/3 = -1/6

Perpendicular slope = 6

Equation: y - 1.75 = 6(x + 0.5)

y = 6x + 4.75

(c) The locus of point P moves along the plane and intersects the lines m(y – 3) = x + 1 and y = mx, where m is a variable. Find the equation of the locus of point P.

x = m(y - 3) - 1

y = m(x) = m(m(y - 3) - 1)

$y = m^2(y - 3) - m$

$y - m^2y = -3m^2 - m$

$y(1 - m^2) = -3m^2 - m$

$y = (-3m^2 - m)/(1 - m^2)$

5. (a) Calculate the height h given that the points A(2,5), B(h,-4) and C(1,2) are collinear.

   Slope AC = (2 - 5)/(1 - 2) = -3/-1 = 3

   Slope AB = (-4 - 5)/(h - 2) = -9/(h - 2)

   Set slopes equal: -9/(h - 2) = 3

   -9 = 3(h - 2)

   -9 = 3h - 6

   3h = -3

   h = -1

   (b) Determine the equation of a line passing through the point (-4, -4) and parallel to the line 2x + 6y – 9 = 0

   Slope of given line: y = -1/3 x + 3/2

   Parallel line: y + 4 = -1/3(x + 4)

   y = -1/3 x - 16/3

6. (a) State the number of lines of symmetry in each shape:

   (i) Circle = **infinite**

   (ii) Tree = **1**

   (iii) Flying kite = **1**

(iv) Cross shape = **2**

(v) Rectangular mat = **2**

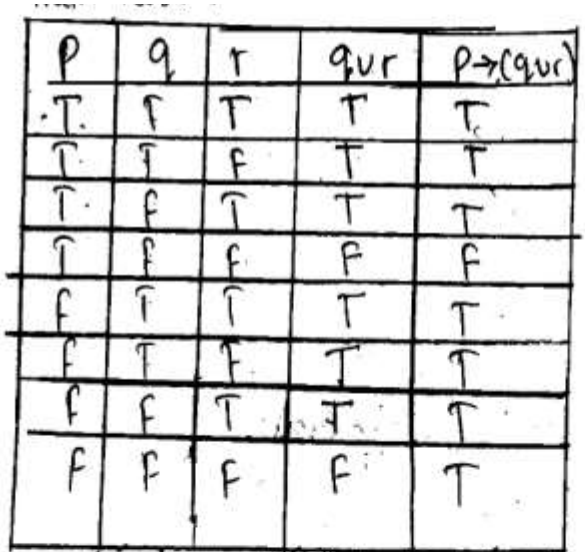
(b) Order of rotational symmetry:

Name of Object	Order of Rotational Symmetry
(i) A rectangular playing card	Two
(ii) A ten thousand Tanzania shillings	Two
(iii) A nonagon	Nine
(iv) A pen	One
(v) A soccer ball	Infinite

7. (a) Given statement: "If 6 is an even number, then it is either divisible by 2 or 4." Represent in symbolic form using p = "6 is even number," q = "6 is divisible by 2," r = "6 is divisible by 4."

   Symbolic form: $p \rightarrow (q \vee r)$

   (b) Truth table



p	q	r	q∨r	p→(q∨r)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

(c) P: 2 + 6 = 8 (true), Q: 6 × 5 = 11 (false)

Symbolic: P ∨ Q

8. (a) Speed L of a particle moving on water is inversely proportional to the cube root of time n. L = 3 when n = 27. Determine L when n = 64.

$L = k / \sqrt[3]{n}$

$3 = k / \sqrt[3]{27} \rightarrow k = 3 \times 3 = 9$

$L = 9 / \sqrt[3]{64} = 9 / 4$

L = 2.25

(b) Determine a when b = 12, given $a \propto (b^2 + 3)$ and a = 4 when b = 5

$a = k(b^2 + 3)$

$4 = k(25 + 3)$

$k = 4 / 28 = 1/7$

b = 12

$a = 1/7 (144 + 3) = 147 / 7$

a = 21

(c) Suppose $p \propto q^2 / r$ and p = 10 when q = 6 and r = 16. Find p when q = 2 and r = 16

$p = k q^2 / r$

$10 = k \times 36 / 16 \rightarrow k = 160 / 36 = 10 / 9$

$p = 10 / 9 \times 4 / 8 = 40 /72$ = **5 / 9**

9. Total sales of six litres of diesel and five litres of petrol = 6000, seven litres diesel + five litres petrol = 6800. Find price per litre using elimination method.

   $6d + 5p = 6000$

   $7d + 5p = 6800$

   Subtract: $d = 800$

   $6(800) + 5p = 6000$

   $4800 + 5p = 6000$

   $5p = 1200$

   $p = 240$

10. $\mu = \{x : 3 \leq x < 18\}$, D = {odd numbers}, P = {primes}, S = {perfect squares}

   (a) Elements:

   μ = {3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17}

   D = {3, 5, 7, 9, 11, 13, 15, 17}

   P = {3, 5, 7, 11, 13, 17}

   S = {4, 9, 16}

   (b) Venn diagram: represent sets D, P, S inside μ

   (c) (i) $P \cap D = \{3, 5, 7, 11, 13, 17\} \cap \{3, 5, 7, 9, 11, 13, 15, 17\}$

   = **{3, 5, 7, 11, 13, 17}**

   (ii) $(P \cup D \cup S)' = \mu - \{3, 4, 5, 7, 9, 11, 13, 16, 17\}$

   = {6, 8, 10, 12, 14}