THE UNINTED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

FORM TWO SECONDARY EDUCATION EXAMINATION, 2004

0041 BASIC MATHEMATICS

Time: 2:30 Hours ANSWERS .

Instructions:

1. this paper consists of section A and B

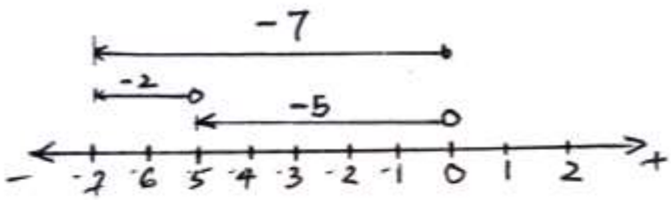
2. Answer all questions

3. Each question carries Four marks.

Prepared by: Maria Marco for TETEA

1. (a) Simplify 14 − [−2 − (8 ÷ 2) + 5].
Solution:
8 ÷ 2 = 4.
Inside the brackets: -2 - 4 + 5 = -1.
14 - [-1] = 14 + 1 = 15.
Answer: 15.

(b) Use the number line to find the sum of (-5) + (-2).
Solution:



-5 + (-2) = -7.

Answer: -7.

2. (a) Arrange 2/5, 5/8, 48%, and 0.6 in ascending order.

Solution:
Convert to decimal:
2/5 = 0.4,
5/8 = 0.625,
48% = 0.48,
0.6 = 0.6.

Ascending order: 0.4, 0.48, 0.6, 0.625.
Answer: 2/5, 48%, 0.6, 5/8.

(b) Decrease 160,000 by 16%.

Solution:
16% of 160,000 = (16/100) × 160,000 = 25,600.
160,000 - 25,600 = 134,400.
Answer: 134,400.

3. (a) Write 3:15 p.m. using a 24-hour clock.

Solution:
3:15 p.m. = 15:15.
Answer: 15:15.

(b) Add:

Kg Hg G
60 9   960
11 0   45

Solution:
Add grams: 960 + 45 = 1005 g (1 kg and 5 g).
Carry 1 kg to kg column.   Add hectograms: 9 + 0 = 9.
Add kilograms: 60 + 11 + 1 = 72.

Answer: 72 kg, 9 hg, 5 g.

4. Express $2.7\dot{9}$ as a fraction in the form a/b.

Solution:
Let $x = 2.7\dot{9}$.
Multiply by 100 to shift repeating digits:
$100x = 279.7\dot{9}$.
Subtract:
$100x - x = 279.7\dot{9} - 2.7\dot{9}$.
$99x = 277$.
$x = 277/99$.
Answer: 277/99.

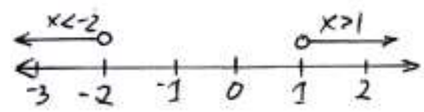
5. Rationalize the denominator of $(p - q) / (\sqrt{q} + \sqrt{p})$.

Solution:
Multiply by the conjugate $(\sqrt{q} - \sqrt{p})$:
$[(p - q)(\sqrt{q} - \sqrt{p})] / [(\sqrt{q} + \sqrt{p})(\sqrt{q} - \sqrt{p})]$.
Simplifying:
Numerator: $(p - q)(\sqrt{q} - \sqrt{p})$.
Denominator: $q - p$.
Answer: $[(p - q)(\sqrt{q} - \sqrt{p})] / (q - p)$.

6. Show on the number line the solution set of the inequality $|2x + 1| > 3$.

Solution:
Solve two cases:
$2x + 1 > 3$ -----> $2x > 2$ --------> $x > 1$.
$2x + 1 < -3$ --------> $2x < -4$ ------> $x < -2$.

Answer: $x < -2$ or $x > 1$.



7. Solve the simultaneous equations:
$1/x + 1/y = 7$,
$2/x + 3/y = 16$.

Solution:
Solving for x and y:
$x = 1/5$, $y = 1/2$.
Answer: $x = 1/5$, $y = 1/2$.

8. Two angles of a pentagon are 58° and 83°. The other three angles are in the ratio 5:6:8. Find the largest angle.
Solution:
Sum of pentagon's angles = $(5 - 2) \times 180 = 540°$.
Remaining sum = $540 - (58 + 83) = 399°$.
Ratios sum = $5 + 6 + 8 = 19$.
Largest angle = $(8/19) \times 399 = 168°$.
Answer: 168°.

9. Use Pythagoras' theorem to find the length x in the right triangle.
Solution:
Given sides: 5m and hypotenuse 13m.
$x^2 + 5^2 = 13^2$.
$x^2 + 25 = 169$.
$x^2 = 144$.
$x = 12$.
Answer: 12m.

10. At what rate should 2340/= be invested for 2 years to earn an interest of 140.4/=?

Solution:
Using the simple interest formula:
$I = PRT / 100$.
$140.4 = (2340 \times r \times 2) / 100$.
Solving for r:
$r = (140.4 \times 100) / (2340 \times 2)$.
$r = 3\%$.
Answer: 3%.

11. (a) Without using tables, evaluate sin 60° / cos 60° × sin 30° / cos 30°.

Solution:
$\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$.
$\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$.
$(\sqrt{3}/2 \div 1/2) \times (1/2 \div \sqrt{3}/2)$.
$(\sqrt{3} \times 2) / (2 \times 1) \times (1 \times 2) / (2 \times \sqrt{3})$.
= 1.
Answer: 1.

(b) Given sin A = 1/2 where A is an acute angle, find the value of $1 - \cos^2 A$.

Solution:
$\sin^2 A + \cos^2 A = 1$.
$\cos^2 A = 1 - \sin^2 A$.
$= 1 - (1/2)^2$.
= 1 - 1/4.
= 3/4.

Answer: 0.75.

12. (a) Simplify (log 8 - log 4) / (log 4 - log 2).

Solution:
$\log 8 = \log (2^3) = 3 \log 2$.
$\log 4 = \log (2^2) = 2 \log 2$.
(3 log 2 - 2 log 2) / (2 log 2 - log 2).
(log 2) / (log 2) = 1.

Answer: 1.

(b) If $N = 2 \times 10^{-8}$, find the value of 1/N in scientific form.

Solution:
$1/N = 1 / (2 \times 10^{-8})$.
$= (10^8 / 2)$.
$= 5 \times 10^7$.
Answer: 5×10^7.

13. Find the equation of the line through the point (2, -2) and crossing the y-axis at the same point as the line y = (5/2)x - 5.

Solution:
The given line crosses the y-axis at -5.
The required line passes through (2, -2) and has equation $y = mx - 5$.
Substituting (2, -2):
$-2 = 2m - 5$.
Solving for m:
$2m = 3$.
$m = 3/2$.
Equation: $y = (3/2)x - 5$.
Answer: $y = (3/2)x - 5$.

14. In a class of 105 students, 25 study Mathematics but not History, 50 study History but not Mathematics. If each student studies at least one subject, determine the number of students who study Mathematics.

Solution:
Students studying both subjects = 105 - (25 + 50) = 30.
Total Mathematics students = 25 + 30 = 55.

Answer: 55.

15. Twice the width of a rectangle is greater than its length by 3 cm. If the perimeter of the rectangle is 36 cm, find its dimensions.
Solution:
Let width = w, length = l.
$2w = l + 3$.
Perimeter = $2l + 2w = 36$.

Solving:
$2w = l + 3$.
$2l + 2w = 36$.

Substituting $2w = l + 3$:
$2l + (l + 3) = 36$.
$3l + 3 = 36$.
$3l = 33$.
$l = 11$.
$w = (11 + 3) / 2 = 7$.

Answer: Width = 7 cm, Length = 11 cm.

16. A cylindrical tank of diameter 140 cm contains water to a height of 2.2 m. Calculate the volume of the water in litres.

Solution:
Convert measurements:
Radius = 140/2 = 70 cm.
Height = 2.2 m = 220 cm.

Volume = $\pi \times r^2 \times h$.
= $\pi \times (70)^2 \times 220$.
= 3,386,636.88 cm^3.

Convert to litres:
3,386,636.88 cm^3 ÷ 1000 = 3,386.64 litres.

Answer: 3,386.64 litres.

17. (a) Transformation T maps the point (x, y) to (x - y, x). Find the image of the point (6, -2) under T.

Solution:
(6 - (-2), 6) = (8, 6).
Answer: (8, 6).

(b) Find the image of point A(2,6) after rotating it through 180°.

Solution:
A 180° rotation about the origin transforms (x, y) to (-x, -y).
Image of (2,6) ------> (-2, -6).

Answer: (-2, -6).

18. Solve for n if $(3/5)^{n-1} = (25/9)^{2n}$.
Solution:
Rewrite in base form:
$(3/5)^{n-1} = (5^2 / 3^2)^{2n}$.
= $5^{4n} / 3^{4n}$.
Equating exponents:
$(3/5)^{n-1} = (3^{-4n} / 5^{4n})$.
Equating powers:
n - 1 = -4n.
5n = 1.
n = 0.2.

19. Given that $x^2 + 8x + Q = (x + K)^2$, find the values of K and Q.

Solution:
Expanding $(x + K)^2$:
$x^2 + 2Kx + K^2 = x^2 + 8x + Q$.
Comparing coefficients:
$2K = 8$, $K = 4$.
$K^2 = Q$, $Q = 16$.
Answer: $K = 4$, $Q = 16$.

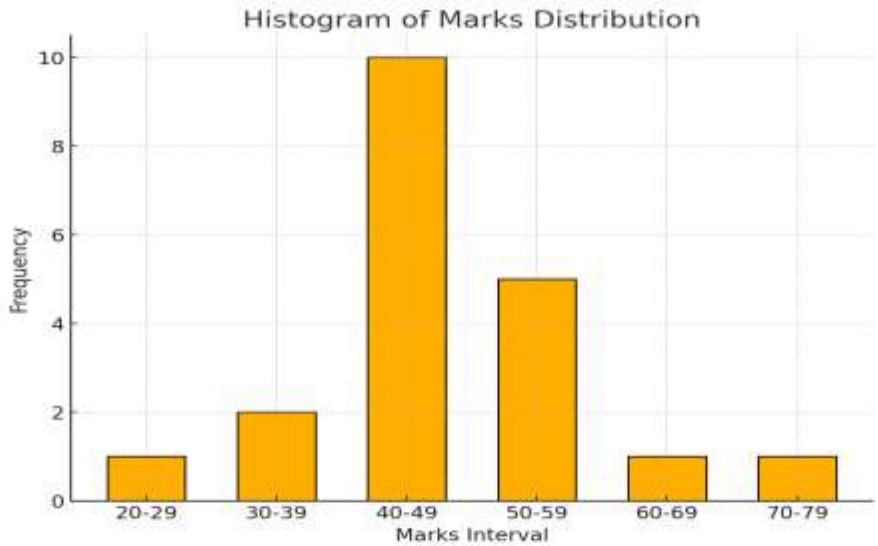
20. The area of a pond on the map is 0.25 cm². If the map has a scale of 1:1000, find the true area in m².
Solution:
Scale factor for area = $(1000)^2 = 1,000,000$.
True area = 0.25 cm² × 1,000,000.
= 250,000 cm².
Convert to m²: 250,000 cm² ÷ 10,000 = 25 m².
Answer: 25 m².

21. (a) In the formula $Tr^2 + (p - k)r + m = 0$, make r the subject.

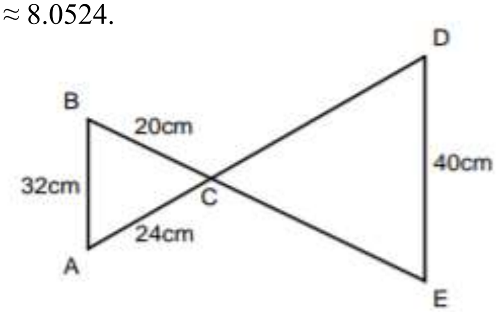
Solution:
This is a quadratic equation in r.
Using the quadratic formula:
$r = (k - p \pm \sqrt{(p - k)^2 - 4Tm}) / (2T)$.

(b) Given that $x + y = 8$ and $x^2 + y^2 = 40$, find the value of xy.
Solution:
Using the identity:
$(x + y)^2 = x^2 + y^2 + 2xy$.
Substituting given values:
$8^2 = 40 + 2xy$.
$64 = 40 + 2xy$.
$2xy = 24$.
$xy = 12$.
Answer: $xy = 12$.

22. In a mathematics test the following marks were obtained:
27, 57, 57, 40, 70, 48, 59, 60, 42, 44, 47, 44, 44, 59, 35, 48, 43, 52, 36, 48
(a) Group the marks in class intervals 20 – 29, 30 – 39, etc. and then construct the frequency distribution table.



23. (a) Show that log(√x³) = 3.3508 if log x ≈ 8.0524.



Solution:
Using logarithmic properties:
log(√x³) = log($x^{3/2}$).
= (3/2) log x.
= (3/2) × 8.0524.
= 3.3508.

Answer: 3.3508.
(b) Use mathematical tables to evaluate (22.8 × √0.0727) / 0.916, correct to 3 significant figures.

Solution:
√0.0727 ≈ 0.2696.
(22.8 × 0.2696) / 0.916.
= 6.711.
Answer: 6.711.

24. (a) Prove that ΔABC is similar to ΔEDC.

Solution:
Two triangles are similar if they have corresponding angles equal or corresponding sides proportional.

- Given that corresponding sides are proportional:
  BC/EC = AB/ED = AC/AD.
- Also, ∠ABC = ∠EDC (corresponding angles).

Since two angles are equal and sides are proportional, ΔABC ~ ΔEDC by the AA similarity theorem.

Answer: Proved by AA similarity theorem.

(b) Calculate the length of EC and CD.
Solution:
Using similarity ratios:
- For EC:
  $BC/EC = AB/ED$.
  $32/EC = 40/24$.
  $EC = (32 \times 24) / 40$.
  $EC = 19.2$ cm.

- For CD:
  $BC/CD = AC/AD$.
  $32/CD = 24/20$.
  $CD = (32 \times 20) / 24$.
  $CD = 26.67$ cm.

Answer:
- $EC = 19.2$ cm.
- $CD = 26.67$ cm.

25. Draw the graph of $y = -x^2 + 4x - 5$ and use it to solve the equation $y = 0$
Solution:
To find the x-intercepts, solve:
$-x^2 + 4x - 5 = 0$
Using the quadratic formula: