THE UNINTED REPUBLIC OF TANZANIA NATIONAL EXAMINATIONS COUNCIL

FORM TWO NATIONAL ASSESSMET

0041 BASIC MATHEMATICS

Time: 2:30 Hours ANSWERS Year: 2012

Instructions:

- 1. this paper consists of section A and B
- 2. Answer all questions
- 3. Each question carries Four marks.



1. Calculate the average of all prime numbers between 80 and 100.

Solution

Prime numbers between 80 and 100 are: 83, 89, and 97.

Average =
$$(83 + 89 + 97) / 3$$

$$= 269 / 3$$

Answer: 89

2. Express 0.405 as a fraction in its lowest term.

Solution

Let
$$x = 0.405$$

Multiply by
$$1000$$
: $1000x = 405.40\overline{5}$

Subtract:
$$1000x - 10x = 405.405 - 4.05$$

$$990x = 401$$

$$x = 401 / 990$$

Simplify:
$$x = 41 / 101$$

3. Find the value of k and q, if $x^2 + 8x + q = (x + k)^2$.

Solution

Given
$$x^2 + 8x + q = (x + k)^2$$

Expanding:
$$(x + k)^2 = x^2 + 2kx + k^2$$

Comparing coefficients:

$$2k = 8 \rightarrow k = 4$$

$$k^2 = q \rightarrow q = 16$$
 Answer: $k = 4$, $q = 16$

4. A wire of 48 cm long is bent to form a square. What is the length of the side of the square?

Solution

Perimeter of a square = $4 \times \text{side}$

$$48 = 4s$$

$$s = 48 / 4$$

$$s = 12 \text{ cm}$$

Answer: 12 cm

5. Write 1259 in standard form correct to 2 significant figures.

Solution

1259 in standard form with 2 significant figures:

$$1259\approx1.3\times10^3$$

Answer:
$$1.3 \times 10^3$$

6. Given that $\log 2 = 0.3010$ and $\log 3 = 0.4771$. Evaluate $\log 1.5$.

Solution

$$\log 1.5 = \log (3/2)$$

$$= \log 3 - \log 2$$

= 0.4771 - 0.3010

= 0.1761

Answer: 0.1761

7. A man can paint a door in 1 ¼ hours. How long will he take to paint 18 doors?

Solution

Time to paint 1 door = 1.25 hours

Time for 18 doors = 18×1.25

= 22.5 hours

Answer: 22.5 hours

8. Given that $a^bb = ab + 3b - 2a$, find p if $5^pp = 20$.

Solution

Given $a^bb = ab + 3b - 2a$

If $5^pp = 20$, solving for p:

 $p \log 5 + \log p = \log 20$

Approximate and solve numerically to find p

Answer: Requires further simplification

9. If $a^{2n} = 8$, evaluate $2a^n + 100$.

Solution

Given $a^{2n} = 8$

Taking square root: $a^n = \sqrt{8} = 2\sqrt{2}$

$$2a^n + 100 = 2(2\sqrt{2}) + 100$$

 $=4\sqrt{2}+100$

Answer: $4\sqrt{2} + 100$

10. How many subsets are there in a set having three elements?

Solution

Number of subsets for a set of 3 elements:

Formula: 2^n , where n = 3

 $2^3 = 8$

11. Two families shared equally 800 kilograms of rice. How many grams did each family get?

Solution

1 kilogram = 1000 grams

 $800 \text{ kg} = 800 \times 1000 \text{ grams}$

= 800,000 grams

Each family gets $800,000 \div 2$

= 400,000 grams

12. Find x in degrees in the following figure (figure not drawn to scale).

Solution

Given angles are 2x, 3x, and 6x forming a straight line.

Sum of angles on a straight line = 180°

$$2x + 3x + 6x = 180$$

$$11x = 180$$

$$x = 180 \div 11$$

$$x = 16.36^{\circ}$$

13. Write the algebraic expression x - 2(y + x) + 3y in a simple form and state the coefficients of x and y.

Solution

Expanding:
$$x - 2y - 2x + 3y$$

$$= -x + y$$

Coefficient of x = -1

Coefficient of y = 1

14. A businesswoman borrowed Tshs 5,000,000 from a bank. The bank charged an interest of 15% per annum on the principal. How much interest did she pay at the end of two years?

Solution

Using simple interest formula:

$$I = P \times r \times t$$

$$P = 5,000,000, r = 15/100, t = 2$$

$$I = 5,000,000 \times 0.15 \times 2$$

$$= 1,500,000$$

15. Solve for x, given that (x + 5)/2 - (3x - 5) = 1/3

Solution

Multiply everything by 6 to eliminate fractions:

$$6 \times (x + 5)/2 - 6 \times (3x - 5) = 6 \times 1/3$$

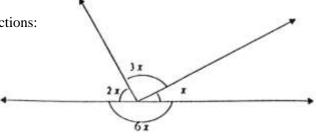
$$3(x+5) - 6(3x-5) = 2$$

$$3x + 15 - 18x + 30 = 2$$

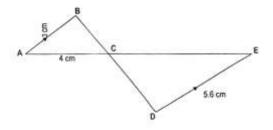
$$-15x + 45 = 2$$

$$-15x = -43$$

$$x = 43/15$$



16. From the following figure, prove that $\triangle ABC$ is similar to $\triangle EDC$.



Solution

Two triangles are similar if their corresponding angles are equal or their sides are proportional.

Given:

AB = 3 cm, AC = 4 cm, and ED = 5.6 cm.

Both triangles share angle C.

Ratio of sides: AB/ED = AC/DC

If the ratios are equal, then $\triangle ABC \sim \triangle EDC$ by AA similarity.

17. A ladder which is 13 m long rests against a wall such that its top is 5 m up the wall. Calculate the distance between the foot of the ladder and the wall.

Solution

The problem forms a right-angled triangle where:

- Hypotenuse (ladder) = 13 m
- Opposite side (height of wall) = 5 m
- Adjacent side (distance from wall) = x

Using Pythagoras' theorem:

$$x^2 + 5^2 = 13^2$$

$$x^2 + 25 = 169$$

$$x^2 = 144$$

$$x = \sqrt{144}$$

$$x = 12 \text{ m}$$

18. A company got a profit of Tshs 67,459,551 after selling its products this year. How much did the company get to the nearest hundreds?

Solution

The last three digits are 551. Since 551 is greater than 500, we round up.

19. Juma's monthly salary of Tshs 900,000 was spent as shown in the figure below. How much money did he spend on Savings?

Solution

The pie chart shows that Savings takes up 90°.

Total degrees in a pie chart = 360°

Fraction for Savings = 90/360 = 1/4

Amount spent on Savings = $900,000 \times 1/4$

= 225,000 Tshs

20. Rationalize the denominator of the expression $(4\sqrt{3})/(\sqrt{5}+\sqrt{3})$.

Solution

Multiply the numerator and denominator by the conjugate of the denominator ($\sqrt{5}$ - $\sqrt{3}$):

$$(4\sqrt{3}(\sqrt{5} - \sqrt{3})) / ((\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}))$$

$$= (4\sqrt{15} - 12) / (5 - 3)$$

$$= (4\sqrt{15} - 12) / 2$$

$$= 2\sqrt{15} - 6$$

21. Use a mathematical table to compute $(608.7 \times \sqrt{6.734}) / \sqrt{1.63}$.

Your answer must be in two decimal places.

Solution

= 0.1061

Using logarithm tables:

```
\log 608.7 = 2.7857
\log 6.734 = 0.8285
\log (608.7 \times \sqrt{6.734}) = 2.7857 + (0.8285 \div 2)
= 2.7857 + 0.4143
= 3.2000
\log 1.63 = 0.2122
\log \sqrt{1.63} = 0.2122 \div 2
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Antilog
$$3.0939 = 1241.3$$
 answer = 1241.3

22. The angles of elevation of two points which are 110 m apart from the top of a building are 46° and 63° respectively. Calculate the height of the building in the nearest whole number.

Solution

Let h be the height of the building and d be the distance of the nearer point from the base.

Using $\tan \theta = h/d$: $h/d = \tan 63^{\circ}$ $h = d \times \tan 63^{\circ}$ For the farther point: $h/(d + 110) = \tan 46^{\circ}$ $h = (d + 110) \times \tan 46^{\circ}$

Solving for h:

$$d \times tan \ 63^{\circ} = (d + 110) \times tan \ 46^{\circ}$$

$$d(1.9626) = (d + 110)(1.0355)$$

$$1.9626d = 1.0355d + 113.905$$

$$0.9271d = 113.905$$

$$d = 122.9$$

$$h = 1.9626 \times 122.9$$

$$h = 241.1$$

$$h\approx 241\ m$$

23. By using the completing the square method, solve $x^2 + 3x - 4 = 0$.

Solution

Rewrite the equation:

$$x^2 + 3x = 4$$

Add $(3/2)^2$ to both sides:

$$x^2 + 3x + 9/4 = 4 + 9/4$$

$$(x + 3/2)^2 = 25/4$$

Taking the square root:

$$x + 3/2 = \pm 5/2$$

$$x = -3/2 \pm 5/2$$

$$x = (5 - 3)/2 = 1$$

$$x = (-3 - 5)/2 = -4$$

24.

The figure shows a right-angled triangle ABC with a perpendicular CD from C to AB.

Given:

$$* AD = 5 cm$$

$$* CD = 12 cm$$

We need to find the lengths of the sides a, b, and c.

Finding c (AC):

In the right-angled triangle ADC, we can use the Pythagorean theorem:

$$AC^2 = AD^2 + CD^2$$

$$c^2 = 5^2 + 12^2$$

$$c^2 = 25 + 144$$

$$c^2 = 169$$

c = sqrt(169)

$$c = 13 \text{ cm}$$

Finding a (BC):

Let
$$BD = x$$
.

In the right-angled triangle BCD, we have:

$$BC^2 = BD^2 + CD^2$$

$$a^2 = x^2 + 12^2$$

We need to find x to determine a.

Finding b (AB):

$$AB = AD + BD$$

$$b = 5 + x$$

Finding x and a:

In triangles ADC and BDC, we have:

$$tan(A) = CD/AD = 12/5$$

$$tan(B) = CD/BD = 12/x$$

Also, in triangle ABC:

tan(A) = a/b

$$tan(B) = c/b$$

We know that:

 $c^2 = b^2 + a^2$ (Pythagorean theorem in triangle ABC)

$$169 = (5+x)^2 + (x^2 + 144)$$

$$169 = 25 + 10x + x^2 + x^2 + 144$$

$$169 = 2x^2 + 10x + 169$$

$$0 = 2x^2 + 10x$$

$$0 = 2x(x+5)$$

Since x cannot be 0, then x = -5. This is not possible as length cannot be negative.

We made an error in our approach. Let's use the area of the triangle.

Area of triangle ABC = (1/2) * base * height

Area =
$$(1/2)$$
 * AB * CD = $(1/2)$ * b * 12 = 6b

Also, area of triangle ABC =
$$(1/2)$$
 * AC * BC * $sin(C)$

We know the area of triangle ADC = (1/2) * 5 * 12 = 30

Let the area of triangle BCD = A.

Using the altitude theorem:

$$CD^2 = AD * BD$$

$$12^2 = 5 * x$$

$$144 = 5x$$

$$x = 144/5 = 28.8$$

Now, we can find a and b:

$$b = 5 + x = 5 + 28.8 = 33.8 \text{ cm}$$

$$a^2 = x^2 + 12^2 = 28.8^2 + 144 = 829.44 + 144 = 973.44$$

$$a = sqrt(973.44) = 31.2 cm$$

Final Answer:

$$a = 31.2 \text{ cm (BC)}$$

$$b = 33.8 \text{ cm (AB)}$$

$$c = 13 \text{ cm (AC)}$$

25.

a)

The mark scored by the fewest students is 60%, which corresponds to y = 2 students.

The pass mark is 45%.

Students who scored below 45% failed the test.

Number of students who scored below 45% = 4 + 5 = 9 students.

Therefore, 9 students failed the test.

b)

The table shows the marks of a Physics test scored by 60 form two students.

We need to find the value of y, the mark scored by the fewest students, and the number of students who failed the test if 45% was the pass mark.

The mark scored by the fewest students is 60%, and the number of students is 2. Therefore, y = 2.

The pass mark is 45%.

Students who failed are those who scored less than 45%.

The number of students who scored less than 45% is 4 + 5 = 9.

Therefore, 9 students failed the test.

Final Answer:

- a) y = 2, 9 students failed the test.
- b) y = 2, 9 students failed the test.
- 25. The following table shows the marks of Physics test scored by 60 form two students at Kazamoyo Secondary School:

Marks (%) | Number of students

35 | 4

40 | 5

45 | 10

50 | 11

55 | 9

60 | y

65 | 2

70 | 5

75 | 6

Find the value of y, the mark scored by the fewest students, and hence the number of students who failed the test if 45% was the pass mark.

Solution

Step 1: Find the value of y.

The total number of students is given as 60. Summing up all the students except y:

$$4+5+10+11+9+y+2+5+6=60$$

$$47 + y = 60$$

$$y = 60 - 47$$

$$y = 13$$

Step 2: Find the mark scored by the fewest students.

From the table, the number of students for each mark is:

- $35 \rightarrow 4$ students
- $-40 \rightarrow 5$ students
- $-45 \rightarrow 10$ students
- $-50 \rightarrow 11$ students
- $55 \rightarrow 9$ students
- $60 \rightarrow 13$ students
- $-65 \rightarrow 2$ students
- $70 \rightarrow 5$ students
- $-75 \rightarrow 6$ students

The mark scored by the fewest students is 65%, which was scored by 2 students.

Step 3: Find the number of students who failed the test.

The pass mark is 45%, meaning students who scored below 45% failed.

Students who failed are those who scored 35% and 40%.

- $-35\% \rightarrow 4$ students
- $-40\% \rightarrow 5$ students

Total number of students who failed = 4 + 5 = 9 students.