THE UNINTED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

FORM TWO NATIONAL ASSESSMET

0041 BASIC MATHEMATICS

Time: 2:30 Hours ANSWERS Year: 2018

Instructions:

1. this paper consists of section A and B
2. Answer all questions
3. Each question carries Four marks.

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1. (a) A block is cut into equal units of 10 g, 20 g, and 35 g. Use prime factorization method to find the smallest possible mass of the block from which the pieces can be cut.

Prime factorization:
$10 = 2 \times 5$
$20 = 2^2 \times 5$
$35 = 5 \times 7$
The least common multiple (LCM) is found by taking the highest power of each prime factor:

LCM = $2^2 \times 5 \times 7 = 4 \times 5 \times 7 = 140$ g
Final answer: 140 g

(b) Evaluate 0.864 + 0.0246 giving your answer correct to 2 significant figures.

$0.864 + 0.0246 = 0.8886$
Rounded to 2 significant figures: 0.89
Final answer: 0.89

2. (a) Find out which of the two fractions, 5/7 or 6/9, is greater.

Convert both fractions to a common denominator:

LCM of 7 and 9 is 63.
$5/7 = (5 \times 9) / (7 \times 9) = 45/63$
$6/9 = (6 \times 7) / (9 \times 7) = 42/63$
Since $45/63 > 42/63$, then 5/7 is greater than 6/9.
(b) If 0.125 of all students in a mixed class are girls, what percentage of the students are boys?

Percentage of girls = $0.125 \times 100 = 12.5\%$
Percentage of boys = $100\% - 12.5\% = 87.5\%$
Final answer: 87.5%

3. (a) Subtract:

m	dm	cm	mm
10	9	31	2
- 8	9	38	9

Step by step subtraction:
Since 31 cm is smaller than 38 cm, borrow 1 dm (10 cm), making 31 cm → 41 cm.
Since 2 mm is smaller than 9 mm, borrow 1 cm (10 mm), making 2 mm → 12 mm.

Now subtract:

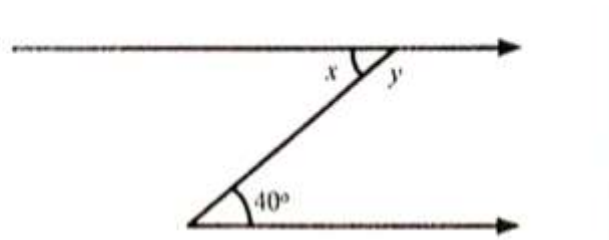
m   dm cm mm
10 8   41   12
 8   9   38   9
----
 2   9   3   3

Final answer: 2 m 9 dm 3 cm 3 mm
(b) Find the simple interest on sh. 10,000,000 invested for 5 years at the rate of 6% per annum.

Simple interest formula:
$SI = (P \times R \times T) / 100$

$= (10,000,000 \times 6 \times 5) / 100$
$= (300,000,000) / 100$
$= 3,000,000$
Final answer: 3,000,000 shillings

4. (a) Calculate the size of the angles X and y in the following figure:
Solution:
Since $x + y = 180°$
But , $x = 40°$, then
$40° + y = 180°$
$y = 140°$
$x = 40°$



(b) (b) Find the perimeter of a right-angled triangle whose base is $(4-\sqrt{2})$ cm and height is $(4+\sqrt{2})$ cm.

Solution:
In a right-angled triangle, the perimeter is calculated by adding the lengths of the base, height, and hypotenuse. The hypotenuse can be found using the Pythagorean theorem:

Hypotenuse = $\sqrt{(\text{base}^2 + \text{height}^2)}$

Base = $(4 - \sqrt{2})$ cm
Height = $(4 + \sqrt{2})$ cm

First, square the base and the height:
$\text{Base}^2 = (4 - \sqrt{2})^2 = 16 - 8\sqrt{2} + 2 = 18 - 8\sqrt{2}$
$\text{Height}^2 = (4 + \sqrt{2})^2 = 16 + 8\sqrt{2} + 2 = 18 + 8\sqrt{2}$

Now, add the squares of the base and height:
$\text{Base}^2 + \text{Height}^2 = (18 - 8\sqrt{2}) + (18 + 8\sqrt{2}) = 36$

Thus, the hypotenuse = $\sqrt{36} = 6$ cm.

Now, the perimeter is the sum of the base, height, and hypotenuse:
Perimeter = $(4 - \sqrt{2}) + (4 + \sqrt{2}) + 6 = 8 + 6 = 14$ cm.

Final Answer:
The perimeter of the triangle is 14 cm.

5(a) Solve:
$2x + y = 20$
$x = 35 - 3y$
by the elimination method.

Solution:
To solve using the elimination method, we aim to eliminate one of the variables by adding or subtracting the equations.
First, substitute the expression for x from the second equation into the first equation:
$2(35 - 3y) + y = 20$
Now, distribute 2:
$70 - 6y + y = 20$
Simplify the equation:
$70 - 5y = 20$
Subtract 70 from both sides:
$-5y = 20 - 70$
$-5y = -50$
Divide both sides by -5:
$y = -50 / -5$
$y = 10$
Now substitute $y = 10$ into the second equation to find x:
$x = 35 - 3(10)$
$x = 35 - 30$
$x = 5$
Final Answer:
$x = 5$, $y = 10$.

(b) Solve the equation $4(p + 1)(1 - p) = 3$.

Solution:
First, expand the left side of the equation:
$4(p + 1)(1 - p) = 3$
Distribute 4:
$4(p + 1)(1 - p) = 4[p(1 - p) + 1(1 - p)]$

$= 4[p - p^2 + 1 - p]$
$= 4[-p^2 + 1]$
$= -4p^2 + 4$
Now, set the equation equal to 3:
$-4p^2 + 4 = 3$
Subtract 4 from both sides:
$-4p^2 = 3 - 4$
$-4p^2 = -1$
Divide by -4:
$p^2 = 1/4$
Take the square root of both sides:
$p = \pm 1/2$

Final Answer:
$p = 1/2$ or $p = -1/2$.

6. (a) The slope of the straight line through the points (7, 4) and (-2, k) is 1, find the value of k.
Solution:
The formula for the slope of a line through two points (x_1, y_1) and (x_2, y_2) is:
slope $= (y_2 - y_1) / (x_2 - x_1)$
Here, the points are (7, 4) and (-2, k). The slope is given as 1, so:
$1 = (k - 4) / (-2 - 7)$
Simplify the denominator:
$1 = (k - 4) / (-9)$
Now multiply both sides by -9:
$-9 = k - 4$
Solve for k:
$k = -9 + 4$
$k = -5$
Final Answer:
$k = -5$.
(b) By using a sketch, find the image of the point (5, 2) under a reflection in the line y = 0, followed by another reflection in the line y = x.

Solution:

- First reflection in the line $y = 0$:
  Reflection in the line $y = 0$ means reflecting the point across the x-axis. The point (x, y) after reflection in $y = 0$ becomes (x, -y).
  For the point (5, 2), the reflection across the x-axis gives (5, -2).

- Second reflection in the line $y = x$:
  Reflection in the line $y = x$ swaps the x and y coordinates. The point (x, y) after reflection in $y = x$ becomes (y, x).

For the point (5, -2), the reflection across the line $y = x$ gives (-2, 5).

Final Answer:
The image of the point (5, 2) under the two reflections is (-2, 5).

7. (a) Use laws of exponents to simplify $(2r^3)^2 / (2r)^3$

Solution:
Using the laws of exponents:

1. Apply the power of a product rule: $(ab)^n = a^n b^n$
   $(2r^3)^2 = 2^2 \times (r^3)^2 = 4 \times r^6$

2. Apply the power of a product rule again for the denominator:
   $(2r)^3 = 2^3 \times r^3 = 8 \times r^3$

Now, simplify the expression:
$(2r^3)^2 / (2r)^3 = (4 \times r^6) / (8 \times r^3)$

Now, simplify the constants and exponents:
$(4 / 8) \times (r^6 / r^3) = 1/2 \times r^{(6-3)} = 1/2 \times r^3$

Final Answer:
The simplified expression is $r^3 / 2$.

(b) If log 2 = 0.3010, log 3 = 0.4771, and log 7 = 0.8451, find log 42.

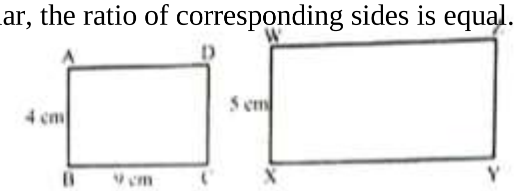
Solution:
We can use the property of logarithms:
$\log(ab) = \log a + \log b$

First, express 42 as the product of primes:
$42 = 2 \times 3 \times 7$

Now, apply the logarithm property:
$\log 42 = \log(2 \times 3 \times 7)$
$\log 42 = \log 2 + \log 3 + \log 7$
Now, substitute the given values:
$\log 42 = 0.3010 + 0.4771 + 0.8451$
$\log 42 = 1.6232$
Final Answer: $\log 42 = 1.6232$.

8. (a) Rectangle ABCD is similar to rectangle WXYZ. If BC = 9 cm, AB = 4 cm, and WX = 5 cm, calculate the length of XY.

Solution:
Set up a proportion: Since the rectangles are similar, the ratio of corresponding sides is equal. We can write the proportion:



AB / WX = BC / XY

Plug in the given values:

4 / 5 = 9 / XY

Solve for XY:

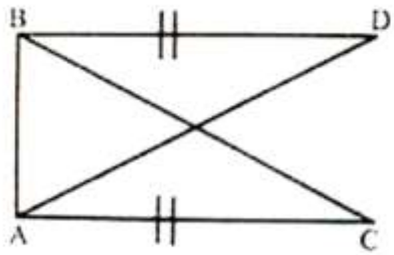
4 x XY = 5 x 9

4 x XY = 45

XY = 45 / 4

XY = 11.25 cm

(b) The figure below shows that AC = BD. Prove that ∠ACB = ∠ADB.



Proof:
- Given: AC = BD
- Common Side: Notice that CD is a common side to both triangles ΔABC and ΔABD.
- Consider Triangles ΔABC and ΔABD:

AB = AB (Common side)

BC = AD (Since ABCD is a rectangle, opposite sides are equal)

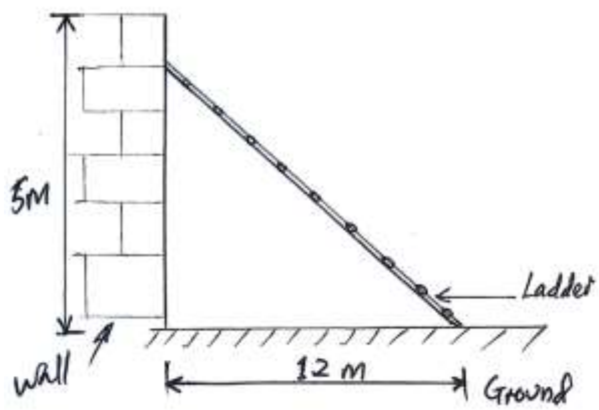
AC = BD (Given)

- SSS Congruence: By the Side-Side-Side (SSS) congruence criterion, triangle ΔABC is congruent to triangle ΔABD.
- Corresponding Angles: Since the triangles are congruent, their corresponding angles are equal.

Therefore, ∠ACB = ∠ADB.
Therefore, we have proven that ∠ACB = ∠ADB.

9. (a) A ladder on the ground leans against a vertical wall whose height is 5 metres. The ground distance between the ladder and the wall is 12 metres.

(i) Draw a diagram to represent this information.
Solution:



(ii) Using the diagram in part (i), find the length of the ladder.

Solution: We can use the Pythagorean theorem to find the length of the ladder. The Pythagorean theorem states that in a right-angled triangle, the square of the length of the hypotenuse (c) is equal to the sum of the squares of the lengths of the other two sides (a and b):

$a^2 + b^2 = c^2$

In this case, a = 5 metres (height of the wall) and b = 12 metres (ground distance). We need to find c (the length of the ladder).

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$169 = c^2$

Now, take the square root of both sides to find c:

$c = \sqrt{169}$ c = 13 metres

So, the length of the ladder is 13 metres.

9. (b) Given that sin A = 3/5 where A is an acute angle, find without using mathematical tables the values of:

(i) cos A

Solution:

We know that $\sin^2 A + \cos^2 A = 1$ (Pythagorean identity).

Given sin A = 3/5, we can substitute it into the equation:

$(3/5)^2 + \cos^2 A = 1$

$9/25 + \cos^2 A = 1$

$\cos^2 A = 1 - 9/25$

$\cos^2 A = 25/25 - 9/25$

$\cos^2 A = 16/25$

Now, take the square root of both sides:

$\cos A = \sqrt{(16/25)}$
$\cos A = 4/5$

So, $\cos A = 4/5$.

(ii) tan A

Solution:
We know that $\tan A = \sin A / \cos A$.

Substitute the values of sin A and cos A:

$\tan A = (3/5) / (4/5)$
$\tan A = 3/4$

So, $\tan A = 3/4$.

(iii) $(1 - \sin A) / (1 - \cos A)$

Solution:
Substitute the values of sin A and cos A:

$(1 - \sin A) / (1 - \cos A) = (1 - 3/5) / (1 - 4/5)$
$= (2/5) / (1/5)$
$= 2$

So, $(1 - \sin A) / (1 - \cos A) = 2$.

10. (a) In a class of 32 students, 18 play golf, 16 play piano, and 7 play both golf and piano. Use a formula to find the number of students who play neither golf nor piano.

Solution:
Let:
- G be the set of students who play golf,
- P be the set of students who play piano,
- N be the number of students who play neither sport.

We can use the formula for the union of two sets to find the number of students who play either golf or piano:

$|G \cup P| = |G| + |P| - |G \cap P|$

Where:
- |G| = 18 (students who play golf),
- |P| = 16 (students who play piano),
- $|G \cap P| = 7$ (students who play both golf and piano).

$|G \cup P| = 18 + 16 - 7 = 27$ students who play either golf or piano.

Now, the total number of students is 32, so the number of students who play neither sport is:
N = Total students - $|G \cup P|$
N = 32 - 27
N = 5 students who play neither golf nor piano.
10. (b) A survey was done among students in a certain school in order to find the most popular subject. In this survey, each student voted once and the results were as follows:

Subject	Mathematics	English	Biology	History	Geography	Physics
Number of Pupils	50	80	120	40	80	30

Show this information in a pie chart.
Solution:
To construct a pie chart, we need to calculate the percentage of students who voted for each subject. First, calculate the total number of students:

Total students = 50 + 80 + 120 + 40 + 80 + 30 = 400 students.

Now, calculate the percentage for each subject:

- Mathematics: (50 / 400) x 100 = 12.5%
- English: (80 / 400) x 100 = 20%
- Biology: (120 / 400) x 100 = 30%
- History: (40 / 400) x 100 = 10%
- Geography: (80 / 400) x 100 = 20%
- Physics: (30 / 400) x 100 = 7.5%