THE UNINTED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

FORM TWO NATIONAL ASSESSMET

0041 **BASIC MATHEMATICS**

Time: 2:30 Hours **ANSWERS** **Year: 2024.**

Instructions:

1. this paper consists of section A and B
2. Answer all questions
3. Each question carries Four marks.

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1. (a) Find the GCF and LCM of the numbers 90 and 240.

Solution:

To find the GCF:
- Prime factorization of 90: $90 = 2 \times 3^2 \times 5$
- Prime factorization of 240: $240 = 2^4 \times 3 \times 5$

The GCF is the product of the smallest powers of common prime factors:
$GCF = 2 \times 3 \times 5 = 30$

To find the LCM:
- The LCM is the product of the highest powers of all prime factors:
$LCM = 2^4 \times 3^2 \times 5 = 720$

Final Answer:
GCF = 30, LCM = 720

(b) Estimate the value of $8108 \div 37$.

Solution:
Estimate 8108 as 8100 and 37 as 40.
$8100 \div 40 = 202.5$
Final Answer:
Approximately 203.

2. (a) Simplify $3\ 9/10 \div (3\ 3/5 - 1\ 1/2)$.

Solution:
Step 1: Convert all mixed fractions to improper fractions:
$3\ 9/10 = 39/10$, $3\ 3/5 = 18/5$, and $1\ 1/2 = 3/2$.

Step 2: Simplify the subtraction in the denominator:
$18/5 - 3/2 = (36/10 - 15/10) = 21/10$.

Step 3: Perform the division:
$39/10 \div 21/10 = (39/10) \times (10/21) = 39/21 = 13/7 = 1\ 6/7$.
Final Answer:
$1\ 6/7$.

(b) Change 1.23 into a mixed number.
Solution:
Let $x = 1.232323...$ (repeating).

Step 1: Multiply both sides by 100 to remove the repeating part:
100x = 123.232323...
Step 2: Subtract the original equation from the new one:
100x - x = 123.232323... - 1.232323...
99x = 122.
Step 3: Solve for x:
x = 122 / 99.

Step 4: Convert to a mixed number:
122 ÷ 99 = 1 remainder 23.

x = 1 23/99.
Final Answer:
1 23/99.

3.(a) How much money will you have to lend in order to get the interest of sh. 36,000 at 5% per annum if you lend it for 6 months?

Solution:
The formula for simple interest is:
Interest = Principal × Rate × Time
Here:
Interest = 36,000
Rate = 5% = 0.05
Time = 6 months = 6/12 = 0.5 years
Rearranging the formula to find Principal:
Principal = Interest / (Rate × Time)
Substitute the values:
Principal = 36,000 / (0.05 × 0.5)
Principal = 36,000 / 0.025
Principal = 1,440,000
Final Answer:
1,440,000 shillings.

(b) Nyanjara bought 50 bottles of milk for 70 children. If the capacity of each bottle is 300 ml, find the amount of milk in litres that Nyanjara bought.

Solution:
The capacity of one bottle is 300 ml. The total milk in millilitres is:
50 × 300 = 15,000 ml.
Convert millilitres to litres:
15,000 ml = 15 litres.

5. (a) Solve for x in the equation $1 - (x + 2)/2 = x - 3$.
Solution:
Start with the given equation:
$1 - (x + 2)/2 = x - 3$.
Step 1: Eliminate the fraction by multiplying through by 2:
$2(1) - (x + 2) = 2(x - 3)$.
Simplify:
$2 - x - 2 = 2x - 6$.
Step 2: Combine like terms:
$- x = 2x - 8$.
Step 3: Solve for x:
$- x - 2x = -8$,
$- 3x = -8$.
Divide by -3:
$x = 8/3$.
(b) What term must be added to $n^2 + 1\frac{1}{2}n = 0$ in order to make the equation a perfect square?
Solution:
To make the equation a perfect square, we need to complete the square.
Step 1: Rewrite $1\frac{1}{2}$ as an improper fraction:
$1\frac{1}{2} = 3/2$.
Step 2: Identify the coefficient of n, which is 3/2. Divide this coefficient by 2 and square the result:
$(3/2 \div 2)^2 = (3/4)^2 = 9/16$.
Thus, the term that must be added is 9/16.

4. (a) Two complementary angles are such that one angle is twice the other. Find the size of those angles.

Solution:
Let the smaller angle be x. Since the angles are complementary, their sum is 90°.
The other angle is 2x.
Equation:
$x + 2x = 90$.
Simplify:
$3x = 90$.

Solve for x:
$x = 90 \div 3 = 30$.
The two angles are:
Smaller angle = 30°.
Larger angle = $2x = 2 \times 30 = 60°$.

(b) The perimeter of an isosceles triangle is 15 cm. If the base is 7 cm long, represent this information in a diagram and then find the length for each of the remaining equal sides.

Solution:
The perimeter of a triangle is the sum of all its sides. For an isosceles triangle:
Perimeter = base + 2 × equal sides.
Let the length of each equal side be x. The equation is:
$7 + 2x = 15$.
Solve for x:
$2x = 15 - 7$,
$2x = 8$,
$x = 8 \div 2 = 4$.
The length of each equal side is 4 cm.

6. (a) A point P'(0, 0) is the image of P(-2, 2) under a translation T. What is the image of the point (5, -1) under the same translation?

Solution:
To find the translation vector, subtract the coordinates of P from P':
Translation vector = (0 - (-2), 0 - 2) = (2, -2).

Apply the same translation to the point (5, -1):
New point = (5 + 2, -1 - 2) = (7, -3).

Final Answer:
The image of (5, -1) is (7, -3).
6(b) If a straight line passes through the point (1, 2) and cuts the y-axis at the point (0, 2), find its equation.

Solution:
The general equation of a straight line is:
$y = mx + c$, where m is the slope and c is the y-intercept.

From the point (0, 2), the y-intercept $c = 2$.
Find the slope (m) using the formula:
$m = (y_2 - y_1) / (x_2 - x_1)$,
$m = (2 - 2) / (0 - 1)$,
$m = 0 / -1 = 0$.

Thus, the equation of the line is:
$y = 2$.

Final Answer:
The equation is $y = 2$.

7(a) If $x = \sqrt{3}$, $y = \sqrt{2}$, and $z(x - y) = 2$, express z in the form $z = a(\sqrt{b} + \sqrt{c})$.
Solution:
Start with the given equation:
$z(x - y) = 2$.
Substitute the values of x and y:
$z(\sqrt{3} - \sqrt{2}) = 2$.

Solve for z:
$z = 2 / (\sqrt{3} - \sqrt{2})$.

Rationalize the denominator:
$z = 2(\sqrt{3} + \sqrt{2}) / [(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})]$,
$z = 2(\sqrt{3} + \sqrt{2}) / (3 - 2)$,
$z = 2(\sqrt{3} + \sqrt{2}) / 1$,
$z = 2(\sqrt{3} + \sqrt{2})$.

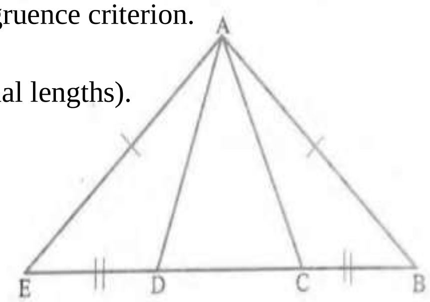
Final Answer:
$z = 2(\sqrt{3} + \sqrt{2})$.

(b) Simplify (log 8 - 2 log 4) / (log 4 - log 2).
Solution:
Step 1: Apply the logarithmic properties.
$\log 8 = \log (2^3) = 3 \log 2$,
$\log 4 = \log (2^2) = 2 \log 2$.
Substitute these values into the expression:
$(\log 8 - 2 \log 4) / (\log 4 - \log 2)$
$= (3 \log 2 - 2 \times 2 \log 2) / (2 \log 2 - \log 2)$.
Step 2: Simplify the numerator and denominator.
Numerator: $3 \log 2 - 4 \log 2 = -\log 2$,
Denominator: $2 \log 2 - \log 2 = \log 2$.
Expression becomes:
$(-\log 2) / (\log 2) = -1$.
8(a) If ΔPQR and ΔLMN are similar, find $\angle QRP$ given that $\angle MNL = 40°$ and $\angle QPR = 60°$.

Solution:
In similar triangles, corresponding angles are equal. Since the angles in a triangle add up to 180°:
$\angle QRP = 180° - (\angle QPR + \angle PQR)$.

Substitute the given values:
$\angle QRP = 180° - (60° + 40°)$,
$\angle QRP = 180° - 100°$,
$\angle QRP = 80°$.

(b) Use the following figure to prove that $\Delta ABC \cong \Delta EDC$:
Solution:
To prove $\Delta ABC \cong \Delta EDC$, we will use the SAS (Side-Angle-Side) congruence criterion.



Step 1: Identify the corresponding sides and angles:
- In the diagram, it is given that AC = AE and AB = EB (marked by equal lengths).
- The angle $\angle BAC$ is common to both triangles ΔABC and ΔEDC.

Step 2: Apply the SAS criterion:
- Side AC = Side AE (Given),
- Side AB = Side EB (Given),
- Angle $\angle BAC = \angle EAD$ (Common).

Since two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle, we conclude that $\Delta ABC \cong \Delta EDC$ by SAS congruence.
Final Answer:
$\Delta ABC \cong \Delta EDC$.

9(a) A student walks from home to school, first eastwards to a road junction 14 km from home, then southwards to school. If the shortest distance from home to school is 20 km, how far is the school from the road junction? Express your answer correct to 3 decimal places.

Solution:

This forms a right triangle where:
- The distance eastwards is 14 km,
- The shortest distance (hypotenuse) is 20 km,
- The distance southwards (school to road junction) is the unknown.

Using the Pythagorean theorem:
$a^2 + b^2 = c^2$,
$14^2 + b^2 = 20^2$,
$196 + b^2 = 400$,
$b^2 = 400 - 196$,
$b^2 = 204$,
$b = \sqrt{204}$.

Simplify:
b = 14.283 km (to 3 decimal places).

Final Answer:
14.283 km.

9(b) Find the value of $2 \sin 60° + \cos 30° - \tan 60°$. Give your answer in radical form.

Solution:

Step 1: Use trigonometric values:
$\sin 60° = \sqrt{3} / 2$,
$\cos 30° = \sqrt{3} / 2$,
$\tan 60° = \sqrt{3}$.

Step 2: Substitute into the expression:
$2 \sin 60° + \cos 30° - \tan 60°$
$= 2(\sqrt{3} / 2) + \sqrt{3} / 2 - \sqrt{3}$,
$= \sqrt{3} + \sqrt{3} / 2 - \sqrt{3}$.

Step 3: Combine terms:
$= (2\sqrt{3} / 2) + (\sqrt{3} / 2) - \sqrt{3}$,
$= (2\sqrt{3} + \sqrt{3}) / 2 - \sqrt{3}$,
$= 3\sqrt{3} / 2 - \sqrt{3}$.

Step 4: Combine further:
$= (3\sqrt{3} - 2\sqrt{3}) / 2$,
$= \sqrt{3} / 2$.

Final Answer:
$\sqrt{3} / 2$.

10(a) The following figure shows the number of elements in each subset. If the number of elements in set A is equal to the number of elements in set B, find $n(A \cup B)$ and $n(A)$.

Solution:
Set A has elements:
$n(A) = (7 - 3x) + x = 7 - 2x$.
Set B has elements:
$n(B) = (5 - x) + x = 5$.
Given $n(A) = n(B)$:
$7 - 2x = 5$,
$-2x = 5 - 7$,
$-2x = -2$,
$x = 1$.
Find $n(A \cup B)$:
$n(A \cup B) = (7 - 3x) + (5 - x) + x = 7 - 3(1) + 5 - 1 + 1$,
$n(A \cup B) = 7 - 3 + 5 - 1 + 1 = 9$.
Find $n(A)$:

$n(A) = 7 - 2x = 7 - 2(1) = 5$.
Final Answer:
$n(A \cup B) = 9$, $n(A) = 5$.

(b) The given pie chart represents the time spent by John in doing different activities on every Monday.

(i) How many hours does he spend for private study?
(ii) How many hours does he sleep?

Solution:

The total time in a day is 24 hours. A full circle in the pie chart is 360°. To find the time spent on an activity, use the formula:
Time for activity = (Angle for activity / 360) × 24.

(i) For private study:
Angle = 45°.
Time = (45 / 360) × 24 = 3 hours.

(ii) For sleeping:
Angle = 105°.
Time = (105 / 360) × 24 = 7 hours.