SMZ

ZANZIBAR EXAMINATIONS COUNCIL

FORM THREE ENTRANCE EXAMINATION

0041 **MATHEMATICS**

Time: 2:30 Hours **ANSWERS** **Year: 2014**

Instructions:

1. this paper consists of section A and B
2. Answer all questions in Section A and Four questions in section B
3. Use a blue or black pen.

1. i) List down all factors of 16:
Factors of 16 are: 1, 2, 4, 8, 16.

ii) List down all factors of 24:
Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24.

Hence, the greatest common factor (GCF) of 16 and 24:
Common factors are: 1, 2, 4, 8.
GCF = 8.

b) i) Round off each of the numbers 8.7, 69.5, 210.11, and 146.8 to the nearest unit:
- 8.7 rounds to 9.
- 69.5 rounds to 70.
- 210.11 rounds to 210.
- 146.8 rounds to 147.

ii) Approximate the value of the numerical expression:
Expression: (146.8 x 210.11) ÷ (69.5 x 8.7)
Rounded values: (147 x 210) ÷ (70 x 9)
Approximation: (30870) ÷ (630) = 49.

2. a) Simplify (3½ + 4⅞) x 2½:
Convert to improper fractions:
- 3½ = 7/2, 4⅞ = 39/8, 2½ = 5/2.
Add 7/2 + 39/8:
- Find LCM of 2 and 8, which is 8.
- (28/8) + (39/8) = 67/8.
Multiply (67/8) x (5/2):
- (67 x 5) ÷ (8 x 2) = 335/16 = 20 15/16.

b) i) Increase 75 by 8 percent:
8% of 75 = (8/100) x 75 = 6.
Increase = 75 + 6 = 81.

ii) Decrease 12½ by 12 percent:
Convert 12½ to a decimal: 12.5.
12% of 12.5 = (12/100) x 12.5 = 1.5.
Decrease = 12.5 - 1.5 = 11.

3. a) Perform the following operations:

i) (5 km + 50 m + 3000 cm) + (2 km + 25 m + 500 cm):
Convert all to meters:

- 5 km = 5000 m, 50 m = 50 m, 3000 cm = 30 m.
- 2 km = 2000 m, 25 m = 25 m, 500 cm = 5 m.

Add: (5000 + 50 + 30) + (2000 + 25 + 5) = 5080 + 2030 = 7110 meters.

3. a) i) Perform the following operations:
(5 km + 50 m + 3000 cm) + (2 km + 25 m + 500 cm), giving your answer in meters.

Convert all distances to meters:
- 5 km = 5000 m, 50 m = 50 m, 3000 cm = 30 m.
- 2 km = 2000 m, 25 m = 25 m, 500 cm = 5 m.

Add: (5000 + 50 + 30) + (2000 + 25 + 5) = 5080 + 2030 = 7110 meters.

Final answer: 7110 meters.

ii) (3.5 liters + 500 milliliters) - (1.8 liters + 700 milliliters), giving your answer in milliliters.

Convert all quantities to milliliters:
- 3.5 liters = 3500 ml, 500 milliliters = 500 ml.
- 1.8 liters = 1800 ml, 700 milliliters = 700 ml.

Calculate: (3500 + 500) - (1800 + 700) = 4000 - 2500 = 1500 milliliters.

Final answer: 1500 milliliters.

b) A person changed 450 US dollars and obtained 765,000 T. shillings. What was the exchange rate in T. shillings per dollar?
Exchange rate = Total amount in T. shillings ÷ Amount in dollars.
Exchange rate = 765,000 ÷ 450 = 1700 T. shillings per dollar.
Final answer: 1700 T. shillings per dollar.

4. a) In the figure provided, find the size of the angles marked by the letters x and y.

Using angle properties:
- At point R, the angle x and the 62° angle form a straight line. The sum of angles on a straight line is 180°.
x = 180° - 62° = 118°.

- At point Y, the angles y and x form a vertically opposite pair of angles. Vertically opposite angles are equal.
y = x = 118°.

b) A rectangular garden of length 24 meters has an area of 240 square meters.
i) Determine its width:

Area of a rectangle = Length x Width.
240 = 24 x Width.
Width = 240 ÷ 24 = 10 meters.

ii) Determine the length of its diagonal:
The diagonal forms the hypotenuse of a right triangle with the length and width as the other two sides.
Using the Pythagorean theorem:
$Diagonal^2 = Length^2 + Width^2$.
$Diagonal^2 = 24^2 + 10^2$.
$Diagonal^2 = 576 + 100 = 676$.
Diagonal = $\sqrt{676}$ = 26 meters.

5. a) What number must be added to the expression $x^2 + 6x + 7$ to make it a perfect square?

The general form of a perfect square trinomial is $(x + a)^2 = x^2 + 2ax + a^2$.
For $x^2 + 6x + 7$:
- 2a = 6, so a = 3.
- $a^2 = 3^2 = 9$.

To make $x^2 + 6x + 7$ a perfect square, add 9 - 7 = 2.
b) Ali is now 12 years younger than her sister Masha. The ratio of their ages three years ago was 1:3.
i) Translate the above statement into a mathematical equation.
Let Ali's current age be x and Masha's current age be y.
Given:
- x = y - 12.
- Three years ago, their ages were x - 3 and y - 3.
- The ratio of their ages three years ago was (x - 3) : (y - 3) = 1:3.

This gives the equation: (x - 3)/(y - 3) = 1/3.

ii) By solving the resulting equation, find their present ages.
Substitute x = y - 12 into (x - 3)/(y - 3) = 1/3:
((y - 12) - 3)/(y - 3) = 1/3.
(y - 15)/(y - 3) = 1/3.
Cross-multiply: 3(y - 15) = 1(y - 3).
3y - 45 = y - 3.
2y = 42.
y = 21.
Substitute y = 21 into x = y - 12:
x = 21 - 12 = 9.

c) Solve for t: 4t - 2(5 - t) = 8 - 3(t + 1).

Expand both sides:

$4t - 10 + 2t = 8 - 3t - 3$.
$6t - 10 = 8 - 3 - 3t$.
$6t - 10 = 5 - 3t$.
$9t = 15$.
$t = 15/9 = 5/3$.

Final answer: $t = 5/3$.

6. a) Rationalize the denominator and simplify:

$\sqrt{6} / (\sqrt{6} - \sqrt{3})$.
Multiply numerator and denominator by the conjugate of the denominator $(\sqrt{6} + \sqrt{3})$:
$(\sqrt{6} / (\sqrt{6} - \sqrt{3})) \times ((\sqrt{6} + \sqrt{3}) / (\sqrt{6} + \sqrt{3})) = (\sqrt{6}(\sqrt{6} + \sqrt{3})) / ((\sqrt{6})^2 - (\sqrt{3})^2)$.
Simplify:
Numerator: $\sqrt{6} \times \sqrt{6} + \sqrt{6} \times \sqrt{3} = 6 + \sqrt{18} = 6 + 3\sqrt{2}$.
Denominator: $(\sqrt{6})^2 - (\sqrt{3})^2 = 6 - 3 = 3$.
Simplified result: $(6 + 3\sqrt{2}) / 3 = 2 + \sqrt{2}$.
Final answer: $2 + \sqrt{2}$.

b) Obtain the values of x and y such that $2^{b-y} = 16$ and $3^{x-y} = 9$.

From $2^{b-y} = 16$:
$16 = 2^4$.
$b - y = 4$.

From $3^{x-y} = 9$:
$9 = 3^2$.
$x - y = 2$.

Solve the system of equations:
1) $b - y = 4$.
2) $x - y = 2$.

Subtract equation 2 from equation 1:
$(b - y) - (x - y) = 4 - 2$.
$b - x = 2$.

Let $b = x + 2$. Substitute into equation 2:
$(x + 2) - y = 4$.
$x + 2 - y = 4$.
$x - y = 2$.
Using substitution, $b = 6$, $x = 4$, $y = 2$.
Final answers: $b = 6$, $x = 4$, $y = 2$.

7. a) Determine the exact values of:

i) $\sqrt{2}$ (cos 45° + sin 45°).

cos 45° = sin 45° = $\sqrt{2}/2$.
$\sqrt{2}$ (cos 45° + sin 45°) = $\sqrt{2}((\sqrt{2}/2) + (\sqrt{2}/2)) = \sqrt{2}(2\sqrt{2}/2) = \sqrt{2} \times \sqrt{2} = 2$.

Final answer: 2.

ii) $2\sqrt{3}$ cos 30° - tan 45°.

cos 30° = $\sqrt{3}/2$, tan 45° = 1.
$2\sqrt{3}$ cos 30° - tan 45° = $2\sqrt{3}(\sqrt{3}/2) - 1 = (2\sqrt{3} \times \sqrt{3})/2 - 1 = 3 - 1 = 2$.
Final answer: 2.

b) If cos P = 15/17, where P is an acute angle, find the value of:

i) sin P.
From the Pythagorean identity: $\sin^2P + \cos^2P = 1$.
$\sin^2P = 1 - \cos^2P = 1 - (15/17)^2 = 1 - 225/289 = 64/289$.
sin P = $\sqrt{(64/289)}$ = 8/17.

ii) tan P.
tan P = sin P / cos P = (8/17) / (15/17) = 8/15.

8. a) Solve for y: $\log_{10}(y + 7) = \log_{10}y + 1$.

Using the properties of logarithms:
$\log_{10}(y + 7) = \log_{10}y + 1$.
Rewrite 1 as $\log_{10}10$:
$\log_{10}(y + 7) = \log_{10}y + \log_{10}10$.
Combine logs on the right side:
$\log_{10}(y + 7) = \log_{10}(10y)$.

Equate the arguments:
y + 7 = 10y.
Solve for y:
7 = 10y - y.
7 = 9y.
y = 7/9.

Final answer: y = 7/9.

b) Find the value of $\log_3((b/a)^2)$ given that log a = 1.83 and log b = 2.73.
Using the property $\log_x((m/n)^k) = k(\log_x m - \log_x n)$:
$\log_3((b/a)^2) = 2(\log_3 b - \log_3 a)$.

Using the change of base formula $\log_x m = \log m / \log x$:
$\log_3 b = \log b / \log 3$, $\log_3 a = \log a / \log 3$.

Substitute the values:
$\log_3 b = 2.73 / \log 3$, $\log_3 a = 1.83 / \log 3$.

$\log_3((b/a)^2) = 2((2.73 / \log 3) - (1.83 / \log 3))$.
$\log_3((b/a)^2) = 2((2.73 - 1.83) / \log 3)$.
$\log_3((b/a)^2) = 2(0.90 / \log 3)$.

Approximate $\log 3 \approx 0.477$:
$\log_3((b/a)^2) = 2(0.90 / 0.477)$.
$\log_3((b/a)^2) \approx 2(1.887)$.
$\log_3((b/a)^2) \approx 3.774$.

Final answer: $\log_3((b/a)^2) \approx 3.774$.

c) Suggest any four ways of reducing industrial pollution in the world.
i) Adopting cleaner production technologies:
Industries should invest in energy-efficient equipment and renewable energy sources to reduce emissions and waste production.
ii) Implementing stricter environmental regulations:
Governments should enforce laws limiting industrial discharge of pollutants and ensure regular monitoring of emissions.
iii) Recycling and reusing industrial waste:
Encouraging the recycling of materials like metals, plastics, and chemicals can significantly reduce waste generation.
iv) Promoting afforestation and green spaces:
Planting trees around industrial areas can help absorb pollutants and improve air quality.

9. b) i) Plot the points A(4, 0), B(0, 3), and the origin (0, 0) on a graph. What is the common name of the resulting shape when these points are joined?

When the points A(4, 0), B(0, 3), and the origin (0, 0) are plotted on a graph and joined, the resulting shape is a right-angled triangle

ii) Calculate the length of the line from A to B.
Use the distance formula:

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
For A(4, 0) and B(0, 3):
Distance = $\sqrt{(0 - 4)^2 + (3 - 0)^2}$.
Distance = $\sqrt{(-4)^2 + 3^2}$.
Distance = $\sqrt{16 + 9}$.
Distance = $\sqrt{25} = 5$.

Final answer: 5 units.

10. a) i) How many subsets are there in a set with three (3) elements?

The number of subsets of a set with n elements is given by 2^n.
For a set with 3 elements:
Number of subsets = $2^3 = 8$.

Final answer: 8 subsets.

ii) List down all subsets of the set S = {a, b, c}.

The subsets of S = {a, b, c} are:
- {} (empty set),
- {a},
- {b},
- {c},
- {a, b},
- {a, c},
- {b, c},
- {a, b, c}.

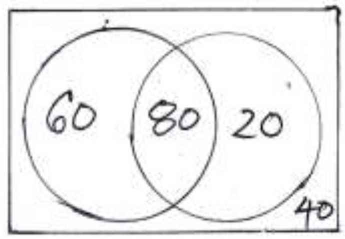
Final answer: {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}.

b) Out of 200 students who appeared in the Form Three Entrance Examination, 140 passed Mathematics and 100 passed Physics. If 40 students failed both subjects, find by using a Venn diagram the number of students who passed:

i) Both subjects
ii) Physics but not Mathematics.

Total students = 200.
Students who failed both = 40.
Students who passed either subject = 200 - 40 = 160.
Let x be the number of students who passed both subjects.
Students who passed Mathematics only = 140 - x.

Students who passed Physics only = 100 - x.
From the Venn diagram:
(Students who passed Mathematics only) + (Students who passed Physics only) + (Students who passed both) = Total students who passed either subject.



$(140 - x) + (100 - x) + x = 160$.
$240 - x = 160$.
$x = 80$.

i) Students who passed both subjects = 80.
ii) Students who passed Physics but not Mathematics = 100 - x = 100 - 80 = 20.

11. a) i) Express the equation $x/(x - 1) = 2(x - 5) + 11$ in the form $x^2 + bx + c = 0$, where b and c are integers.

Start with the given equation:
$x / (x - 1) = 2(x - 5) + 11$.

Multiply through by (x - 1) to eliminate the denominator:
$x = (x - 1)(2(x - 5) + 11)$.

Expand the right-hand side:
$x = (x - 1)(2x - 10 + 11)$.
$x = (x - 1)(2x + 1)$.

Expand further:
$x = 2x^2 + x - 2x - 1$.
$x = 2x^2 - x - 1$.

Rearrange to form $x^2 + bx + c = 0$:
$2x^2 - x - 1 - x = 0$.
$2x^2 - 2x - 1 = 0$.

ii) Solve the resulting equation by the factorization method.

Equation: $2x^2 - 2x - 1 = 0$.

Find two numbers whose product is (2)(-1) = -2 and sum is -2.
Numbers: -2 and 1.

Rewrite the middle term:
$2x^2 - 2x - 1 = 2x^2 - 2x + x - 1 = 0$.

Group terms:

$(2x^2 - 2x) + (x - 1) = 0.$
$2x(x - 1) + 1(x - 1) = 0.$

Factorize:
$(2x + 1)(x - 1) = 0.$

Solve for x:
$2x + 1 = 0$ or $x - 1 = 0.$
$x = -1/2$ or $x = 1.$

Final answers:
$x = -1/2, x = 1.$

b) Given that m * n = ½(m + n) - m, evaluate (9 * 17) * 6.

Step 1: Evaluate 9 * 17.
m * n = ½(m + n) - m.
9 * 17 = ½(9 + 17) - 9.
9 * 17 = ½(26) - 9.
9 * 17 = 13 - 9 = 4.

Step 2: Use the result from Step 1 to evaluate (9 * 17) * 6.
(9 * 17) * 6 = 4 * 6.
4 * 6 = ½(4 + 6) - 4.
4 * 6 = ½(10) - 4.
4 * 6 = 5 - 4 = 1.

Final answer: 1.
2. a) If AB = 3/4 and BC = 5/6, find CA.

Using the property of a triangle, CA = AB + BC.
CA = 3/4 + 5/6.
Find the LCM of 4 and 6, which is 12.
Convert fractions:
3/4 = 9/12, 5/6 = 10/12.
CA = 9/12 + 10/12 = 19/12.

Final answer: CA = 19/12 or 1 7/12.

b) In setting the selling price P of an article, a shopkeeper doubled its cost price. What will be the profit if the selling price is T. Shillings 46,500?

If the selling price is double the cost price, then:

Selling price = 2 × Cost price.
Cost price = Selling price ÷ 2.
Cost price = 46,500 ÷ 2 = 23,250.

Profit = Selling price - Cost price.
Profit = 46,500 - 23,250 = 23,250.

Final answer: Profit = T. Shillings 23,250.

13. a) i) Determine the perimeter of the polygon ABCDE.
From the figure, AB = 4 cm, BC = 3 cm, and the area of rectangle ACDE = 20 m^2.
To find the missing dimensions, use the area of the rectangle:
Area = Length × Width.
20 = AD × DC.

Assume AD = 5 m and DC = 4 m (as it satisfies 5 × 4 = 20).
Perimeter = AB + BC + CD + DE + EA.
Perimeter = 4 + 3 + 5 + 4 + 4 = 20 cm.

ii) What is the name given to this polygon?
The polygon is a pentagon (5 sides).

b) i) Show that triangles KLM and KPN are similar.

Two triangles are similar if their corresponding angles are equal.
From the figure:
- ∠KLM = ∠KPN = 40° (common angle).
- ∠LMK = ∠PNK = 40° (given).

Since two angles are equal, the triangles are similar by AA (Angle-Angle) similarity criterion.

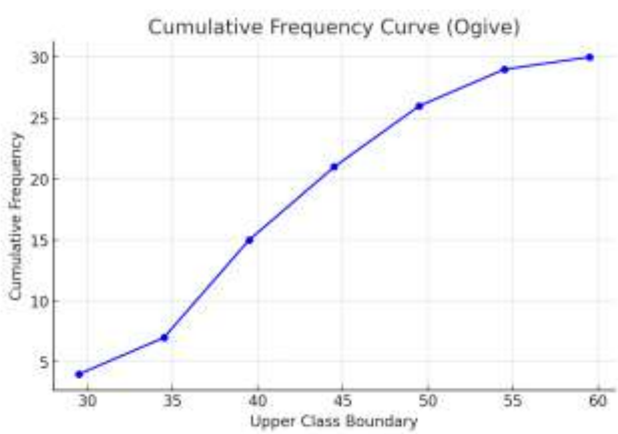
ii) Using similarity property, calculate the length of the side labeled x.
From similarity, the ratio of corresponding sides is equal.
KL / KP = LM / PN.
Substitute known values:
12 / 8 = 7 /x.
Cross-multiply:
12x = 8 × 7.
12x = 56.
x= 56 / 12 = 4 2/3 or 4.67 cm.

14. a) i) Prepare a frequency distribution that includes cumulative frequencies by grouping the ages into class intervals 25–29, 30–34, ..., 55–59.

Class intervals: 25–29, 30–34, 35–39, 40–44, 45–49, 50–54, 55–59.
Ages (data): 35, 46, 41, 59, 55, 47, 38, 32, 27, 35, 52, 45, 40, 56, 49, 53, 33, 42, 39, 36, 43, 42, 53, 41, 43, 29, 32, 25, 38, 44.

Class Interval	Frequency (f)	Cumulative Frequency (CF)
25–29	4	4
30–34	3	7
35–39	8	15
40–44	6	21
45–49	5	26
50–54	3	29
55–59	1	30

ii) Hence draw the cumulative frequency curve (ogive).



b) Ten packets of a chemical are such that five weigh 20.01 g, three weigh 19.98 g, and two weigh 20.03 g. Calculate the mean (average) mass of the packets.

Given weights:
- 5 packets weigh 20.01 g each.
- 3 packets weigh 19.98 g each.
- 2 packets weigh 20.03 g each.

Total mass
(5 × 20.01) + (3 × 19.98) + (2 × 20.03).
= 100.05 + 59.94 + 40.06.

= 200.05 g.
Mean mass
Mean = Total mass ÷ Total number of packets.
Mean = 200.05 ÷ 10 = 20.005 g.
Final answer: Mean mass = 20.005 g.