SMZ

ZANZIBAR EXAMINATIONS COUNCIL

FORM THREE ENTRANCE EXAMINATION

0041 **MATHEMATICS**

Time: 2:30 Hours **ANSWERS** **Year: 2016**

Instructions:

1. this paper consists of section A and B
2. Answer all questions in Section A and Four questions in section B
3. Use a blue or black pen.

1. a) Write 624.3278 correct to
   i. Five significant figures
   Solution:
   624.33
   ii. Three decimal places
   Solution:
   624.328

b) Express 1.86 as an improper fraction in its simplest form.
Solution:
1.86 = 1 + 86/100 = 186/100 = 93/50

2. a) Evaluate without using mathematical tables:
   2log3 + log36 - log9
Solution:
Using logarithmic properties:
2log3 + log36 - log9
= $\log(3^2)$ + log36 - log9
= log9 + log36 - log9
= log(36)
= 1.5563 (approximately)
b) Simplify:
   $(6x^{-4} \times 2x^3) \div 3x^{-3}$
Solution:
$(6 \times 2 \times x^{-4} \times x^3) \div (3 \times x^{-3})$
$= (12x^{-1}) \div (3x^{-3})$
$= 4x^2$

3. a) Rationalize the denominator:
   (√3 + √2) / (√5 + √2)
Solution:
Multiply numerator and denominator by the conjugate of the denominator:
= [(√3 + √2)(√5 - √2)] / [(√5 + √2)(√5 - √2)]
= (√15 - √6 + √10 - 2) / (5 - 2)
= (√15 - √6 + √10 - 2) / 3

3. b) Solve for x if $\sqrt{(3^{x+2})} + 17 = 8$.

Solution:
Step 1: Isolate the square root.
$\sqrt{(3^{x+2})} = 8 - 17$
$\sqrt{(3^{x+2})} = -9$
Squaring both sides, $3^{x+2} = 81$

$3^{x+2} = 3^4$, compare the exponents,
$x + 2 = 4$
$x = 2$.

4. a) Given $\alpha\sqrt{((x^2 - n)/m)} = a^2/b$, make x the subject of the formula.

Solution:
Step 1: Remove the square root by squaring both sides.
$\alpha^2((x^2 - n)/m) = (a^2/b)^2$

Step 2: Simplify the equation.
$(\alpha^2/m)(x^2 - n) = a^4/b^2$

Step 3: Multiply through by m.
$\alpha^2(x^2 - n) = (ma^4)/b^2$

Step 4: Expand and solve for x^2.
$x^2 - n = (ma^4)/(\alpha^2 b^2)$
$x^2 = n + (ma^4)/(\alpha^2 b^2)$

Step 5: Solve for x.
$x = \pm\sqrt{[n + (ma^4)/(\alpha^2 b^2)]}$

b) Given $x = 4.5 \times 10^{-7}$ and $z = 7.2 \times 10^5$, find y in standard form if $z = xy$.
Solution:
$z = xy$
$y = z/x$
$y = (7.2 \times 10^5) / (4.5 \times 10^{-7})$
Step 1: Simplify the coefficients.
$7.2 / 4.5 = 1.6$
Step 2: Simplify the powers of 10.
$10^5 / 10^{-7} = 10^{12}$
Step 3: Combine the results.
$y = 1.6 \times 10^{12}$

5. a) i. The price of one kilogram of sugar is 1500/=, while the price of one kilogram of beans is 1600/=. A person buys x kg of sugar and y kg of beans. Express this as an algebraic expression.

Solution:
Total cost = (1500x) + (1600y)

ii. From the given price above (i), a person buys 3 kg of sugar and 2 kg of beans. What is the total amount of money the person should pay?

Solution:
Total cost = (1500 × 3) + (1600 × 2)
= 4500 + 3200
= 7700/=

b) i. Simplify x - (5 - (2x + 6) - 10).
Solution:
x - (5 - 2x - 6 - 10)
= x - (−2x - 11)
= x + 2x + 11
= 3x + 11

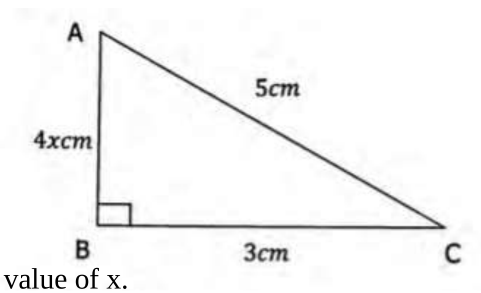
ii. Solve for x: 16 - 2(2x + 3) = x - 11.

Solution:
16 - 4x - 6 = x - 11
10 - 4x = x - 11
10 + 11 = x + 4x
21 = 5x
x = 21/5
x = 4.2

6. i. The right-angled triangle ABC in the diagram has sides of length 4 cm, 3 cm, and 5 cm.

Find the value of x.

Solution:
In the diagram, the hypotenuse is 5 cm, and the other two sides are 4 cm and 3 cm.
Since the triangle is right-angled, Pythagoras' theorem holds:
$c^2 = a^2 + b^2$, where c is the hypotenuse.



$5^2 = 4^2 + 3^2$
25 = 16 + 9
25 = 25

This confirms the given dimensions, and there is no unknown value of x.

ii. Calculate the area of the triangle ABC.

Solution:
The area of a triangle is given by:
Area = 1/2 × base × height

Area = 1/2 × 4 × 3
Area = 6 cm^2

7. a) An observer on the top of a cliff 25 m above sea level views a boat on the sea at an angle of depression of 75°. How far is the boat from the foot of the cliff?

Solution:
Let the distance of the boat from the foot of the cliff be d.
Using the tangent function:
tan(75°) = opposite / adjacent
tan(75°) = 25 / d

Rearranging for d:
d = 25 / tan(75°)

Using tan(75°) = 3.732 (approximately):
d = 25 / 3.732
d ≈ 6.7 m

b) Without using a table, simplify (sin30°cos30°) / tan30°.

Solution:
Using trigonometric values:
sin30° = 1/2, cos30° = √3/2, and tan30° = 1/√3.

(sin30°cos30°) / tan30° = [(1/2)(√3/2)] / (1/√3)
= (√3/4) / (1/√3)
= (√3/4) × (√3/1)
= 3/4

8. a) Find the distance between a point A (2, 7) and B (5, 3).

Solution:
The distance between two points is given by:
$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$

Substitute the values:
$d = \sqrt{[(5 - 2)^2 + (3 - 7)^2]}$
$= \sqrt{[3^2 + (-4)^2]}$
= √[9 + 16]
= √25
= 5

b) The gradient of a line joining (2, 1) and (k, 3) is 4. Find the value of k.

Solution:
The gradient is given by:
$m = (y_2 - y_1) / (x_2 - x_1)$

Substitute the values:
4 = (3 - 1) / (k - 2)
4 = 2 / (k - 2)

Multiply through by (k - 2):
4(k - 2) = 2
4k - 8 = 2
4k = 10
k = 10 / 4
k = 2.5

9. a) i. Write out an extra row for cumulative frequencies.

Solution:

Mark:	0-19	20-39	40-59	60-79	80-100
Frequency:	7	24	83	52	34
Cumulative:	7	31	114	166	200

ii. Draw a cumulative frequency curve.
(This requires graph paper, so it cannot be represented here.)

c) The number of goals scored by Mazengo's football team is given in the table. Draw a bar chart to represent this information.
(This requires graph paper, so it cannot be represented here.)

10. a) Define the following terms.
i. Intersection of two sets:
The intersection of two sets A and B, denoted by A ∩ B, is the set containing all elements that are common to both A and B.
ii. Union of two sets:
The union of two sets A and B, denoted by A ∪ B, is the set containing all elements that belong to either A or B or both.

b) If μ = {a, b, c, d, e, f}, A = {a, b, c}, and B = {e, d}, find:
i. A ∩ B:
A ∩ B = {}
ii. A ∪ B:

$A \cup B = \{a, b, c, d, e\}$

c) In a class of 42 students, 31 study History and 26 study Physics. Using a Venn diagram and otherwise, find the number of students who study Physics only.

Solution:

Let the number of students studying both subjects be x.

Total students = 42

Students studying only History = 31 - x

Students studying only Physics = 26 - x

Using the total:

$(31 - x) + (26 - x) + x = 42$

$57 - x = 42$

$x = 15$

Students studying only Physics = 26 - 15 = 11

11. a) If $x^2 + ax + 4 = 0$ is a perfect square, find the value of a.

Solution:

For a quadratic equation to be a perfect square, its discriminant must be zero.

The discriminant (Δ) is given by:

$\Delta = b^2 - 4ac$

Here, b = a, a = 1, and c = 4. Substitute these values:

$\Delta = a^2 - 4(1)(4)$

$\Delta = a^2 - 16$

Set $\Delta = 0$:

$a^2 - 16 = 0$

$a^2 = 16$

$a = \pm 4$

12. a) State Pythagoras theorem.

Solution:

Pythagoras' theorem states that in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. Mathematically:

$c^2 = a^2 + b^2$

b) A rope of length 18 m is tied to the top of a flagpole. The other end of the rope is fixed to a point 13 m from the base of the flagpole. How high is the flagpole?

Solution:

Using Pythagoras' theorem:

Let h be the height of the flagpole. The hypotenuse is 18 m, and the base is 13 m.

$h^2 + 13^2 = 18^2$

$h^2 + 169 = 324$

$h^2 = 324 - 169$

$h^2 = 155$

$h = \sqrt{155}$

$h \approx 12.45$ m

13. a) A car is bought for 4,000,000/- and sold for 4,500,000/-.

i. Profit

Profit = Selling price - Buying price

Profit = 4,500,000 - 4,000,000

Profit = 500,000/-

ii. Percentage profit

Percentage profit = (Profit / Buying price) × 100

Percentage profit = (500,000 / 4,000,000) × 100

Percentage profit = 12.5%

b) A loan was made at the rate of 8% for 6 months. If the interest charged was 40,000/=, find the amount borrowed.

Solution:

Simple interest formula:

$I = P \times r \times t$

Where:

I = Interest = 40,000

P = Principal (amount borrowed)

r = Rate = 8% = 0.08

t = Time in years = 6 months = 0.5

Substitute the values:

$40,000 = P \times 0.08 \times 0.5$

$40,000 = P \times 0.04$

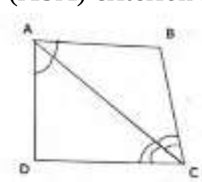
$P = 40,000 / 0.04$

P = 1,000,000/-

14. a) In the diagram below, AC bisects ∠DAB and ∠DCB. Show that ΔADC ≡ ΔABC.

Solution:

To prove that ΔADC ≡ ΔABC, we use the Angle-Side-Angle (ASA) criterion for congruence:



1. AC is a common side for both triangles.
2. ∠DAB = ∠DCB, as given (AC bisects these angles).
3. ∠DAC = ∠CAB (AC bisects ∠DAB).

Since two angles and the included side are congruent in both triangles, by ASA, $\Delta ADC \equiv \Delta ABC$.

b) Solve the pair of simultaneous equations by the elimination method:
$5x + 2y = 14$
$3x - 4y = 24$

Solution:
Step 1: Multiply the equations to make the coefficients of y equal.
Multiply the first equation by 2 and the second equation by 1:
$10x + 4y = 28$
$3x - 4y = 24$

Step 2: Add the two equations to eliminate y:
$(10x + 4y) + (3x - 4y) = 28 + 24$
$13x = 52$
$x = 52 / 13$
$x = 4$

Step 3: Substitute x = 4 into the first equation to solve for y:
$5(4) + 2y = 14$
$20 + 2y = 14$
$2y = 14 - 20$
$2y = -6$
$y = -6 / 2$
$y = -3$

Final solution:
$x = 4, y = -3$