SMZ

ZANZIBAR EXAMINATIONS COUNCIL

FORM THREE ENTRANCE EXAMINATION

0041 **MATHEMATICS**

Time: 2:30 Hours **ANSWERS** **Year: 2021**

Instructions:

1. this paper consists of section A and B

2. Answer all questions in Section A and Four questions in section B

3. Use a blue or black pen.

1. a) Find the LCM and GCF of the following set of numbers: 8, 16, 30.

Solution:
Prime factorization:
$8 = 2^3$
$16 = 2^4$
$30 = 2 \times 3 \times 5$

LCM = Product of the highest powers of all primes:
LCM = $2^4 \times 3 \times 5 = 240$

GCF = Product of the lowest powers of common primes:
GCF = $2^1 = 2$

LCM = 240, GCF = 2

b) Write the next three numbers in the following sequence: 20, 21, 23, 26, ...

Solution:
The differences between consecutive terms are:
21 - 20 = 1
23 - 21 = 2
26 - 23 = 3
The next differences will follow the pattern: 4, 5, 6.

Next numbers:
26 + 4 = 30
30 + 5 = 35
35 + 6 = 41
Next three numbers: 30, 35, 41

2. Evaluate the following:
i) 34564 - 27898
Solution:
34564 - 27898 = 6666

ii) 64.23 × 23
Solution:
64.23 × 23 = 1477.29

iii) (25 × 8) + 50 ÷ 2
Solution:
(25 × 8) + 50 ÷ 2 = 200 + 25 = 225

3. a) Convert $0.3\dot{6}$ to an exact fraction.

Solution:
Let x = 0.363636...
Multiply both sides by 100 to remove the repeating part:
100x = 36.363636...
Subtract the original equation from this:
100x - x = 36.363636... - 0.363636...
99x = 36
Divide both sides by 99:
x = 36 / 99
Simplify the fraction:
36 / 99 = 4 / 11
Exact fraction: 4 / 11

b) A plant of height 80 cm increases in height by 12% after watering. Calculate its height after watering.
Solution:
Increase = 12% of 80 = 0.12 × 80 = 9.6 cm
New height = 80 + 9.6 = 89.6 cm
Height after watering = 89.6 cm

4. a) Juma deposited sh. 3000 in a bank that paid interest at the rate of 8% per annum. How much money was in his account at the end of the two years?
Solution:
The formula for simple interest is:
I = P × R × T

Where:
P = 3000
R = 8% = 0.08
T = 2
I = 3000 × 0.08 × 2
I = 480
Total amount = Principal + Interest
Total = 3000 + 480
Total = 3480

b) Convert the following units:
i) 0.45 kilograms into grams.

Solution:
1 kilogram = 1000 grams
0.45 kilograms = 0.45 × 1000 = 450 grams

ii) 0.006 m^3 into litres.
Solution:
1 m^3 = 1000 litres
0.006 m^3 = 0.006 × 1000 = 6 litres

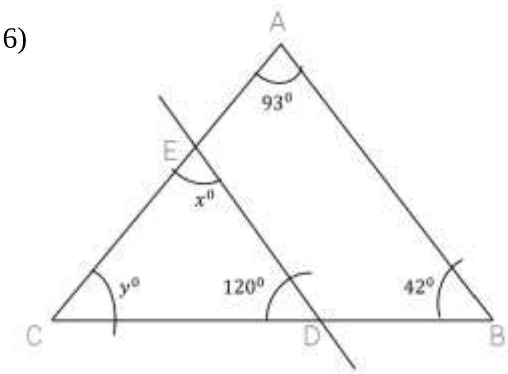
5. Find the value of the angles x° and y° in the diagram.
Solution:
In triangle ABC:
The sum of angles in a triangle is 180°.
$93° + 42° + y° = 180°$
$y = 45°$
in ΔECD,
$x + y + 120° = 180°$
$x + 45° + 120° = 180°$
$x = 15°$

6. a) Simplify $(4m^2n^3) / (2m^{-3}n^4)$.
Solution:
Divide the coefficients:
4 / 2 = 2
For the powers of m:
$m^2 / m^{-3} = m^{2+3} = m^5$
For the powers of n:
$n^3 / n^4 = n^{3-4} = n^{-1} = 1/n$
$= 2m^5 / n$

b) Calculate the value of x in the equation $(x + 4) / 3 + (2x - 1) / 2 = 1 / 6$.

Solution:
Find the least common denominator (LCD) of 3, 2, and 6, which is 6. Multiply through by 6 to eliminate the fractions:

$6 \times [(x + 4) / 3] + 6 \times [(2x - 1) / 2] = 6 \times (1 / 6)$
$2(x + 4) + 3(2x - 1) = 1$
Expand:
$2x + 8 + 6x - 3 = 1$
$8x + 5 = 1$
Solve for x:
$8x = 1 - 5$
$8x = -4$
$x = -4 / 8$
$x = -1/2$



7. a) Evaluate 1/7 ÷ (3/7 ÷ 9/14).
Solution:
Use BODMAS
Simplify the denominator:
3/7 ÷ 9/14 = 2/3
Now evaluate:
1/7 ÷ (2/3) = 1/7 × 3/2 = 3/14
= 3/14

b) Juma is climbing a slope rock face. Every 5 minutes he goes 6 meters forward and slides 2 meters backward. How far does he climb in 30 minutes?

Solution:
Net distance climbed in 5 minutes = 6 - 2 = 4 meters
In 30 minutes, the number of 5-minute intervals is:
30 / 5 = 6
Total distance climbed = 4 × 6 = 24 meters

8. a) The area of a trapezium is 90 cm^2. Its height is 6 cm, and one of the parallel sides is twice the other. Calculate the parallel sides.

Solution:
The formula for the area of a trapezium is:
Area = (1/2) × (a + b) × h

Where:
a = one parallel side
b = the other parallel side
h = height = 6 cm
Area = 90 cm^2

Substitute the given values:
90 = (1/2) × (a + b) × 6
90 = 3(a + b)
a + b = 30

Since one side is twice the other:
Let a = 2b.
Substitute into a + b = 30:
2b + b = 30
3b = 30
b = 10
Substitute b = 10 into a = 2b:

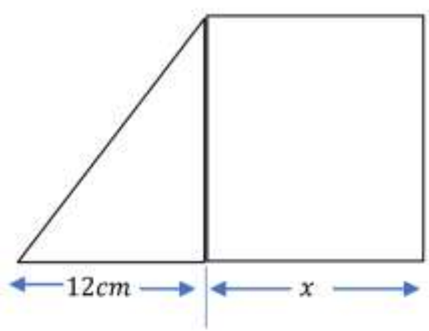
$a = 2 \times 10 = 20$
The parallel sides are 20 cm and 10 cm.

b) Rationalize the denominator of $\sqrt{2} + 1 / (5 - \sqrt{6})$.
Solution:
Multiply numerator and denominator by the conjugate of the denominator:
$(\sqrt{2} + 1) / (5 - \sqrt{6}) \times (5 + \sqrt{6}) / (5 + \sqrt{6})$
Numerator:
$(\sqrt{2} + 1)(5 + \sqrt{6}) = 5\sqrt{2} + \sqrt{12} + 5 + \sqrt{6} = 5\sqrt{2} + 2\sqrt{3} + 5 + \sqrt{6}$
Denominator:
$(5 - \sqrt{6})(5 + \sqrt{6}) = 25 - 6 = 19$

$= (5\sqrt{2} + 2\sqrt{3} + 5 + \sqrt{6}) / 19$

9. a) The following figure is a square of side x cm with a triangle at one end. If the total area is 40 cm^2, calculate the value of x.

Solution:
The area of the square is x^2 .... (i)
The area of the triangle is:
$(1/2) \times \text{base} \times \text{height} = (1/2) \times 12 \times x = 6x$ .... (ii)
Total area:
$x^2 + 6x = 40$
Simplify the quadratic equation:
$x^2 + 6x - 40 = 0$
Solve using the quadratic formula:
$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$
Here, a = 1, b = 6, c = -40:
$x = [-6 \pm \sqrt{(6^2 - 4(1)(-40))}] / 2(1)$
$x = [-6 \pm \sqrt{(36 + 160)}] / 2$
$x = [-6 \pm \sqrt{196}] / 2$
$x = [-6 \pm 14] / 2$



$x = (14 - 6) / 2 = 8 / 2 = 4$ (valid solution as $x > 0$).
x = 4 cm.

b) If $L = 2\pi rh$, write r as the subject of the formula.
Solution:
$L = 2\pi rh$
Divide both sides by $2\pi h$:
$r = L / (2\pi h)$
$r = L / (2\pi h)$.

10. a) If x * y is given by the formula 3x + y, calculate the value of:
i) 3 * 4
Solution:
Using the formula x * y = 3x + y:
3 * 4 = 3(3) + 4 = 9 + 4 = 13
ii) a, if 2 * a = 12

Solution:
Using the formula x * y = 3x + y:
Substitute 2 * a = 12:
3(2) + a = 12
6 + a = 12
a = 12 - 6
a = 6
b) Calculate the values of a, b, and c such that $2x^2 - 8x + 15 = a(x + b)^2 + c$.

Solution:
Expand $a(x + b)^2 + c$:
$a(x^2 + 2bx + b^2) + c = ax^2 + 2abx + ab^2 + c$
Compare coefficients with $2x^2 - 8x + 15$:
For x^2: a = 2
For x: 2ab = -8, substitute a = 2:
2(2)b = -8
4b = -8
b = -2
For the constant term:
$ab^2 + c = 15$, substitute a = 2 and b = -2:
$2(-2)^2 + c = 15$
2(4) + c = 15
8 + c = 15
c = 15 - 8
c = 7
11. a) Asha stands 40 m from the base of a building with height 12 m. The distance from her feet to the top of the building is 44 m. If a is the angle of elevation, calculate:

i) sin a
sin a = opposite / hypotenuse = 12 / 44 = 3 / 11
ii) cos a
cos a = adjacent / hypotenuse = 40 / 44 = 10 / 11
iii) tan a
tan a = opposite / adjacent = 12 / 40 = 3 / 10
sin a = 3 / 11, cos a = 10 / 11, tan a = 3 / 10

b) Given that $\log_5 2 = 0.4307$ and $\log_5 3 = 0.6826$, calculate the value of $\log_5 (8 \div 3)$.
Solution:
$\log_5 (8 \div 3) = \log_5 8 - \log_5 3$
$= \log_5 2^3 - \log_5 3$
$= 3(0.4307) - 0.6826$
$= 0.6095$

12. a) Let translation T_1 and T_2 have points (4,1) and (3, -4) respectively. T_1 takes B(3,4) to B and T_2 takes B to B''.
i) Calculate the coordinate of B.

Solution:
Translation T_1 takes point B(3, 4) to a new position. The translation vector for T_1 is (4, 1). This means to get the new coordinates of point B (B'), we add the translation vector (4, 1) to the coordinates of point B(3, 4):

B' = (3 + 4, 4 + 1)
B' = (7, 5)
So, the coordinates of B' are (7, 5).

ii) Calculate the coordinate of B''.
Solution:
Translation T_2 takes point B' (7, 5) to a new position. The translation vector for T_2 is (3, -4). To get the new coordinates of point B'' (the result of applying T_2), we add the translation vector (3, -4) to the coordinates of point B' (7, 5):
B'' = (7 + 3, 5 - 4)
B'' = (10, 1)
So, the coordinates of B'' are (10, 1).

b) Find the gradient of the line passing through the points (4, 6) and (6, 8).
Solution:
The gradient (m) of a line passing through two points (x_1, y_1) and (x_2, y_2) is calculated using the formula:
$m = (y_2 - y_1) / (x_2 - x_1)$
Substitute the values of the points (4, 6) and (6, 8):

m = (8 - 6) / (6 - 4)
m = 2 / 2
m = 1
So, the gradient of the line is 1.

13. a) Define the following terms on set theorem.

i) Union of two sets: The union of two sets, denoted as A ∪ B, is the set containing all elements that are in set A, set B, or in both. In other words, the union combines all unique elements from both sets.

ii) Intersection of two sets: The intersection of two sets, denoted as A ∩ B, is the set containing only the elements that are common to both sets A and B. In other words, it includes only the elements that exist in both sets.

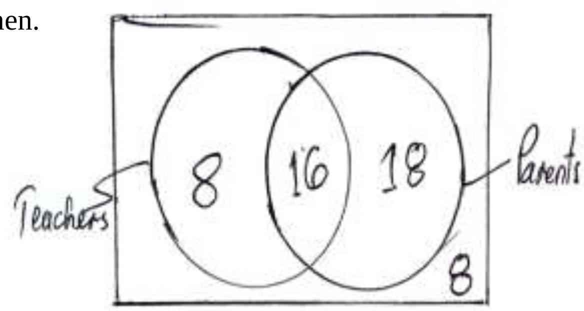
b) There are 50 men at a meeting of whom 24 are teachers, 34 are parents, and 16 are both teachers and parents. By using Venn diagrams, find the number of men who are neither teachers nor parents.

Solution:

Let the total number of men at the meeting be represented by the universal set, U, with 50 men.

Let:

- Set T represent the men who are teachers.
- Set P represent the men who are parents.



We are given the following:

- $|T| = 24$ (the number of teachers)
- $|P| = 34$ (the number of parents)
- $|T \cap P| = 16$ (the number of men who are both teachers and parents)

To find the number of men who are neither teachers nor parents, we need to find the number of men who are in neither set T nor set P. We can use the principle of inclusion-exclusion:

$|T \cup P| = |T| + |P| - |T \cap P|$

Substitute the given values:

$|T \cup P| = 24 + 34 - 16$

$|T \cup P| = 42$

Now, the number of men who are neither teachers nor parents is the complement of the union of T and P in the universal set U:

Number of men who are neither teachers nor parents = $|U| - |T \cup P|$

Number of men who are neither teachers nor parents = 50 - 42

Number of men who are neither teachers nor parents = 8

Final answer: 8 men are neither teachers nor parents.

14. a) Define the term Histogram.

Solution:
A histogram is a type of bar graph that represents the frequency distribution of numerical data. The data is grouped into intervals or bins, and the height of each bar represents the frequency of data points within each interval. Histograms are used to visualize the distribution of data and to see patterns such as skewness, normal distribution, and outliers.

b) The following frequency distribution table shows the marks of 100 students in the end of term mathematics examination.

Marks (%)	Frequency
41 - 50	50
51 - 60	60
61 - 70	20
71 - 80	17
81 - 90	18
91 - 100	11

Solution:

i) Write an extra row for the cumulative frequency.
The cumulative frequency is the running total of the frequencies, starting from the first interval and adding each subsequent frequency. To calculate the cumulative frequency, we sum the frequencies progressively.

Marks (%)	Frequency	Cumulative Frequency
41 - 50	50	50
51 - 60	60	50 + 60 = 110
61 - 70	20	110 + 20 = 130
71 - 80	17	130 + 17 = 147
81 - 90	18	147 + 18 = 165
91 - 100	11	165 + 11 = 176

ii) Draw cumulative frequency curve on the graph paper.

To draw the cumulative frequency curve (Ogive), plot the cumulative frequencies against the upper limit of each class interval.